

**ST.XAVIER'S COLLEGE, KOLKATA**  
(Autonomous under Calcutta University)

Department of Mathematics

**Syllabus for**

**THREE YEAR HONOURS DEGREE COURSE IN MATHEMATICS  
AND  
THREE YEAR GENERAL COURSE IN MATHEMATICS**

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**SYLLABUS FOR THE THREE-YEAR HONOURS DEGREE COURSE IN MATHEMATICS**

(Total 8 papers each of 100 marks)

**Course Structure**

**First Year**

**Semester I**

**Paper I:**Module 1:Algebra 1(50 marks)

Module 2: Calculus1(50 marks)

**Semester II**

**Paper II:**Module1: Calculus2(including Differential Equations)and Introduction to Metric Spaces (50 marks)

Module 2: Vector Calculus and Analytic Geometry (50 marks)

**Second Year**

**Paper III:**Module1: Analysis 1(50 marks)

Module2: Algebra 2(50 marks)

**Paper IV:**Module1:Linear Programming and Boolean Algebra(35+15)

Module2:Mechanics 1 (50 marks)

**Third Year**

**Paper V:**Module 1: Probability Theory (40 marks)

Module 2: Numerical Analysis (60 marks)  
Theory and Practical (30+30)

**PaperVII:**Module 1: Algebra 3 (40 marks)

Module 2: : Programming in C  
Theory and Practical (20+40)

**PaperVI:**Module 1 Analysis 2(50 marks)

Module2: Optional 1(50 marks)

**PaperVIII:**Module 1:Differential Equation2

and Complex Analysis(25+25)  
Module 2: Optional 2(50 marks)

**Availability of classes (minimum) per module:**

Year	Sem	No. of weeks available for teaching per semester	No. of Honours classes/week	Total no. of Hons. Classes/sem	No. of module/ sem	No.of classes/ module
I	I	15	18	270	2	135
	II	15	18	270	2	135
II	I	15	18	270	2	135
	II	15	18	270	2	135
III	I	15	28	420	4	105
	II	11	28	308	4	77

## Detailed Syllabus

(Figures in the margin indicate the number of lectures recommended to cover the topic)

### First Year

#### Paper- I

##### Paper I, Module 1 (Algebra I)

50 marks

1. Sets(4), Relations, Equivalence Relations and Partitions (4). Mappings (6). Definition, examples, properties of Groups (8) , groups of  $n$  th roots of unity (statement of De Moivre's Theorem), permutation groups, group of residue modulo classes (3), properties relating to order of an element of a group, order of a group (3), Subgroups(3), Cyclic groups(4), Lagrange's Theorem for finite groups (2).
2. Relations between the roots and coefficients of general polynomial equation in one variable. Transformations of equations. Descartes's Rule of signs—statement and application, Cardan's method of solution of a cubic equation and Ferrari's method of solution of a biquadratic equation. Sturm's theorem (statement only and applications)(2+3+2+1+1+2 = 11)
3. Inequalities—Cauchy-Schwarz, involving A.M., G.M. and H.M., Holder's, Jensen's, Minkowski's—statement and applications. (4)
4. Definition and examples of Vector Spaces. Subspaces. Linear Span. Linear Dependence, independence and their properties. Bases and dimension, computation of bases, Infinite dimensional vector spaces : only examples. (10)  
Linear Transformation and Matrices (10). Elementary operation on matrices. Inverse of a matrix. Rank. Row Reduction. Permutation Matrices. Matrix operations. Symmetric, skew – symmetric, Hermitian and skew- Hermitian matrices(6). System of linear equations. Determinant of a square matrix—properties and problems (4). Invariant subspaces corresponding to a matrix operator and concept of eigenvectors. Eigenvalues, characteristic equation of a square matrix (detailed discussion). Cayley- Hamilton theorem and its use in finding the inverse of a matrix. Diagonalisation of matrices(8).

**Total No. of classes for general teaching:90. No. of tutorial classes:45.**

**Books Recommended:** (1) Algebra—M. Artin

(2) An Introduction to Abstract Algebra—M.K.Sen, S.Ghosh, P.Mukhopadhyay

(3) Linear Algebra—Rao, Bhimasankaram

(4) Topics in Algebra—I.N.Herstein

(5) The Theory of Equations (Vol. I)—Burnside, Panton

(6) Higher Algebra—Barnard and Child.

(7) Modern Algebra—Surjeet Singh and Qazi Zameruddin

(8) First Course in Abstract Algebra—Fraleigh

(9) Abstract Algebra—D.S.Dummit and R.M.Foote

(10) Higher Algebra(Abtract and Linear)—S.K.Mapa

**(General Approach: Emphasis is on methods/ manipulation/application and NOT on theory/analysis/structure)**

1. Introduction to Real Number System.
2. Sequence of real numbers: Definition of bounds of a sequence and monotone sequence. Limit of a sequence:  $\varepsilon$ - $n$  definition of convergence (through examples). Statement of Limit Theorems. Concept of convergence and divergence of monotonic sequences—applications of the theorems, in particular, definition of  $e$ . Statement of Cauchy's General Principle of convergence and its applications. Cauchy's First and Second Limit Theorems (statement only) and their applications (7).
3. Infinite series of constant terms. Convergence and divergence (definitions). Cauchy's Principle as applied to infinite series (applications only). Series of positive terms: Statements of Comparison Test, D'Alembert's Ratio Test, Cauchy's  $n$ th Root Test, Raabe's Test—applications. Alternating Series: Statement of Leibniz Test and its applications(6).
4. Real valued functions defined on an interval: Limit (Cauchy's definition of the limit of a function—examples), Algebra of Limits(statement) (4). Continuity(2), discontinuities of different kinds(2). Acquaintance (no proof) with the important properties of continuous functions on closed intervals(2). Statement of existence of inverse function of a strictly monotone function and its continuity(2).
5. Derivative—its geometrical and physical interpretation. Sign of derivative—monotone increasing and decreasing functions. Relation between continuity and differentiability(4). Successive differentiation(2), Leibnitz's theorem and its applications(3).
6. Statement of Rolle's Theorem and its geometrical interpretation (2). Mean Value Theorems of Lagrange and Cauchy(3). Statements of Taylor's and Maclaurin's Theorems with Lagrange's and Cauchy's form of remainders(1). Taylor's and Maclaurin's infinite series for functions like  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $(1+x)^n$ ,  $\log(1+x)$  with restrictions wherever necessary(3). Indeterminate Forms: L'Hospital's Rule: statement and problems only(2).
7. The principle of maxima and minima (application of Taylor's series) and its applications for a function of a single variable in geometrical, physical and other problems(3).
8. Applications of differential calculus: Tangents and Normals(5). Rectilinear Asymptotes(for Cartesian equation only)(3). Curvature of Plane curves(3). Tests for concavity and convexity(2). Points of inflexion, Multiple points: definitions and examples of singular points(Node, Cusp and Isolated Point)(3). Tracing of curves in Cartesian and polar co-ordinates—methods and problem solving(3). Envelopes of family of straight lines and of curves(problems only), Evolutes(3).
9. Reduction Formulae related to definite/indefinite integrals(brief review of indefinite integrals)(5). Revision of Definite Integrals—properties and examples(4).
10. Definition of Improper Integrals: statement of (1)  $\mu$ -test, (2) comparison test(limit form excluded)—simple problems only. Use of Beta and Gamma functions (convergence and important relations being assumed) (5).

**Total No. of classes for general teaching:90. No. of tutorial classes:45.**

- Books Recommended** (1) Introduction to Real Analysis—Bartle, Sherbert  
(2) Calculus(Vol.I)—T.M.Apostol  
(3) Undergraduate Analysis—S.Lang  
(4) Mathematical Analysis—Shanti Narayan

- (5) Differential Calculus – Shanti Narayan
- (6) Integral Calculus—Shanti Narayan
- (7) A First Course in Real Analysis—S.K.Berberian
- (8) Theory and Applications of Infinite Series—K.Knopp
- (9) Advanced Calculus—D.Widder

## Paper- II

**Paper II, Module 1** (Calculus II and Introduction to Metric Spaces)

**50 marks**

**(General Approach: Emphasis is on methods/ manipulation/application and NOT on theory/analysis/structure)**

1. Functions of two and three variables: geometrical representations. Limit and continuity (definitions only) for functions of two variables. Partial derivatives—knowledge and use of Chain Rule. Exact differentials (emphasis on problem solving only). Functions of two variables—successive partial derivatives: statement of Schwarz’s Theorem on commutativity of mixed derivatives. Euler’s Theorem on homogeneous function of two and three variables. Maxima and minima of functions of not more than three variables—Lagrange’s method of undetermined multiplier—problems only. Implicit function in case of functions of two variables (existence assumed) and derivative. Jacobians for functions of two and three variables—simple properties including functional dependence(12).
2. Ordinary differential equations: significance of ordinary differential equations: geometrical and physical consideration. Formation of differential equation by eliminating arbitrary constants(4). Solution of ordinary differential equation—definition and examples(2). Equations of first order and first degree : statement of existence theorem(1). Exact differential equation of first order—necessary and sufficient conditions for exactness(2). Separable , homogeneous equations(2). Integrating factor, rules of finding integrating factor(3).First order Linear Equations: integrating factor. Equations reducible to the first order linear equations(2). First order higher degree equations solvable for x,y,p. Clairaut’s equation: singular solution and its geometrical meaning. Transformation of a first order differential equation to Clairaut’s form (3).  
Application to geometrical, mechanical and rate problems(4).  
Explicit methods of solving higher order linear differential equations: Basic Theory of linear differential equations(5). Order reduction method(2). Method of undetermined coefficients(3), method of Variation of parameters(2), Symbolic Operator D(2), transformation of the equation by changing the dependent variable/ the independent variable(3).  
Simple eigenvalue problems(3). Simultaneous linear differential equations and their solutions by matrix method(2).Higher order exact O.D.E.
3. Introduction to general Metric Space  $(M, \rho)$  with special reference to Real Metric Space  $(R, d)$  (with the Euclidean metric  $d$ ) : definition and examples—usual metric on  $R, C, R^n$ , discrete metric, sequence space and function space. Topological properties of a metric space—open, closed spheres in a metric space (open, closed intervals in  $(R, d)$ ), interior point, limit point in  $(M, \rho)$  and  $(R, d)$ —examples. Open, Closed sets in  $(M, \rho)$  and  $(R, d)$  – basic

properties and their structures, examples. Closure of a set  $A$  in  $(M, \rho)$  – closure of  $A$  is the smallest closed set containing  $A$ .(30)

**Total No. of classes for general teaching:90. No. of tutorial classes:45.**

**Books Recommended:** (1) Calculus—S.Lang  
(2) Calculus—T.M.Apostol  
(3) Differential Equations—S.L.Ross  
(4) Metric Space—E.T.Copson  
(5) Ordinary Differential Equations--E.Coddington  
(6) Differential Equations with Applications and Historical Notes—G.F.Simmons  
(7) Topology of Metric Spaces—S.Kumaresan

**Paper II , Module 2** (Vector Calculus and Analytic Geometry)

**50 marks**

Introduction to Vector Algebra: scalar and vector product of three vectors. Product of four vectors. Reciprocal vectors.(10)

General equation of second degree in two and three variables (matrix approach) (5)—Pair of straight lines(8) and conics , its tangent, normal and conjugate diameters, Poles and Polars(10). Polar equation of conics(5).

The straight line and the plane(10). Sphere, cone , cylinder(10).

Brief introduction to conicoids(5), tangent planes, normals, generating lines(10). Knowledge of cylindrical, polar and spherical polar co-ordinates, their inter-relation (no deduction required)(2).

Vector differentiation with respect to a scalar variable: vector functions of one scalar variable, derivative of a vector, second derivative of a vector, derivatives of sums and products, velocity and acceleration as derivatives(5).

Concepts of scalar and vector fields, physical significance. Directional Derivative. Gradient. Divergence and curl, Laplacian(10)

Vector integration: Line integrals as integrals of vectors, Circulation, Irrotational vector, Work done, conservative force, Potential orientation. Statements and verification of Theorems of Gauss, Green and Stokes. Calculation of area, volume, and surface area, length of plane curve(15).

**Total No. of classes for general teaching:110. No. of tutorial classes:25.**

**Books Recommended:** (1) Co-ordinate Geometry of three dimensions—J.T.Bell  
(2) Vector Calculus—Spiegel (Schaum)  
(3) Elementary Vector Analysis—C.E.Weatherburn(Vol. I&II)  
(4) Vector Analysis—Louis Brand  
(5) Analytic Geometry—D.Chatterjee  
(6) Vector Analysis—J.C.Tallack  
(7) Vector Analysis with Application to Geometry and Physics—M. Schwartz, S.Green, W.A.Rutledge

## Second Year

**Paper III , Module 1** (Mathematical Analysis with special reference to real analysis)**50 marks**  
[Approach: Analysis, Theory, Structure]

### Part I

#### Set, Relation, Function

Algebra of sets, ordered pair, Cartesian product, function—injective, surjective, bijective, composite functions, Countable, Equivalent sets with usual related results.

For real function of real variable—bounded function, step function, monotone function(2).

#### Real Number System

( $\mathbb{R}, +, \cdot, <$ , Completeness axiom) is a complete ordered field. Completeness through LUB axiom (passing reference to Nested Interval Property as equivalent to the LUB property and also to convergence of Cauchy sequence in  $\mathbb{R}$  as equivalent to the LUB property).

$\mathbb{Q}$  (set of all rational numbers) is countable but not complete.  $\mathbb{R}$  is complete but not countable.  $\mathbb{R}$  (and  $\mathbb{Q}$ ) has Archimedean property.  $\mathbb{Q}, \mathbb{Q}^c$  ( $\mathbb{Q}^c$  denotes the complement of  $\mathbb{Q}$  in  $\mathbb{R}$ ) are dense in  $\mathbb{R}$ . Passing reference to Peano axioms for  $\mathbb{N}$  (set of all natural numbers). To mention that well-ordering of  $\mathbb{N}$  is equivalent to the Mathematical Induction Principle of  $\mathbb{N}$ .

Extended Real Number System (RNS) with the inclusion of  $+\infty, -\infty$  symbols (in relation to the ordered structure of RNS).

Definition and properties of  $|x|, x \in \mathbb{R}$  :

$$(1) |x - y| \geq 0, (2) |x - y| = |y - x|, (3) |x - y| \leq |x - z| + |z - y|$$

( $|x|$  denotes the distance from the origin(10).

#### General Metric Space (MS) with special reference to the real MS (with the Euclidean metric)

Introduction of  $(M, \rho)$  through the example of  $(\mathbb{R}^2, d_2)$  where  $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$  and  $d_2$  is the Euclidean metric on  $\mathbb{R}^2$ .

To mention that same holds for  $(\mathbb{R}^n, d_n)$ .

Examples to illustrate the generality of  $M$  and  $\rho$  --

Example: (1) Discrete MS, (2)  $(\mathbb{R}^2, d_*)$  where  $d_*(x, y) = |x_1 - y_1| + |x_2 - y_2|$ , (3) an example where  $M$  consists of bounded sequences etc. (sequence space), (4) an example where  $M$  consists of bounded functions etc. (function space).

(Link between the last two examples)

Subspace:  $(\mathbb{Q}, d)$  is a subspace of  $(\mathbb{R}, d)$ . (4)

Bounded sets in a MS—two definitions of a bounded set—their equivalence [ $d(x, y) < k$  ;  $d(a, x) < k$ ].

Discrete MS is bounded and  $(\mathbb{R}, d)$  is unbounded.

Example: Let  $(M, \rho)$  be a MS and define  $\rho_1(x, y) = \frac{\rho(x, y)}{1 + \rho(x, y)}$ . To prove that  $(M, \rho_1)$  is a

bounded MS and  $0 \leq \rho_1 \leq \rho$ . (2)

#### Topological Properties of a MS

Open/Closed spheres in a MS (open/closed intervals in  $\mathbb{R}$ ). Interior Point/ Limit Point in  $(M, \rho)$  /  $(\mathbb{R}, d)$ . Open/ Closed sets in  $(M, \rho)$  /  $(\mathbb{R}, d)$ —basic properties and structure of them. Examples of limit

points in  $(\mathbb{R}, d)$ ,  $(\mathbb{R}^2, d_2)$ , Function Space etc. Closure of a set in  $(M, \rho)$ —closure of  $A$  is a closed set. Closure of  $A$  is the smallest closed set  $D$  containing  $A$ .

Diameter of  $A$  ( $A$  is a subset of  $M$ ) with respect to  $(M, \rho)$  – circle diameter as an example of it.

Dense subsets in  $M$ — $D$  is dense iff closure of  $D$  equals  $M$ . Separable MS—examples of separable/non-separable MS. Separable in what sense? Passing reference to the separation concepts in topological spaces(TS)(6).

### Convergence of sequences in general MS/ Real MS and completeness in general MS/Real MS

Convergent Sequences—limit of a sequence—Cauchy sequence—results—

- (a) convergent sequence is a bounded sequence, (b) Cauchy sequence is a bounded sequence, (c) convergent sequence is a Cauchy sequence.

(Cauchy sequence is a convergent sequence in real MS—this will be proved in due course)

Cauchy sequence need not converge in general MS. In  $(\mathbb{Q}, d)$ , there exists Cauchy sequence which does not converge.

Definition of complete MS(CMS) and Examples. Result—

- (a) Let  $(M, \rho)$  be a CMS,  $A$  is a subset of  $M$ ,  $(A, \rho)$  is CMS iff  $A$  is closed in  $(M, \rho)$ .

- (b) Let  $(M, \rho)$  be a CMS ;  $\phi \neq A_n \subseteq M$ ;  $A_n$  is closed set,  $n = 1, 2, 3, \dots$ ;  $A_1 \supseteq A_2 \supseteq \dots$

and  $\lim_{n \rightarrow \infty} d(A_n) = 0$ . Then  $\bigcap_{n=1}^{\infty} A_n$  is a singleton set.

[ If we omit the condition  $\lim_{n \rightarrow \infty} d(A_n) = 0$ , then we have the result  $\bigcap_{n=1}^{\infty} A_n \neq \phi$  .]

NIP for  $\mathbb{R}$  via LUB—note that we have not proved the completeness of  $\mathbb{R}$  via CS(Cauchy sequence). Thus we can not use the above result directly here for  $\mathbb{R}$ .

Also we can prove that NIP implies LUB in  $\mathbb{R}$ . Thus NIP is equivalent to LUB. Later on, we shall prove NIP implies CS- convergence in  $(\mathbb{R}, d)$ . Further, we can show that CS- convergence implies NIP with respect to  $\mathbb{R}$ . Thus, finally, for  $\mathbb{R}$ : LUB, NIP and CS-convergence are equivalent.

NWD (Nowhere dense) sets and Baire Category Theorem— $\mathbb{R}$  is uncountable via Baire Category Theorem.(8)

### Continuity and Compactness

$f: (MS)_1 \rightarrow (MS)_2$ .

Continuity at a point and continuity on a set. Continuity via sequence convergence. Continuity via open sets (passing reference to continuity in TS).

Homeomorphic and isometric spaces—homeomorphism is topological equivalence—isometricism is metrical equivalence.

Uniform continuity—results: Under uniform continuity, Cauchy sequence transforms into Cauchy sequence.

Compactness through FOOSC—finite open subcover. Structure of open sets in a subspace.  $K$  is compact implies  $K$  is closed and bounded ( converse holds in  $(\mathbb{R}, d)$ : Heine-Borel Theorem, will be considered in due course).

Results: (a)  $(M, \rho)$  is compact MS; A subset of  $M$ ,  $A$  is closed implies  $A$  is compact.

- (b) Continuous image of compact MS is compact. Continuous image of  $[a, b]$  under real valued function is closed/bounded and indeed connected, that is ,  $f([a, b]) = [c, d]$ .

[ Proof of compactness of  $[a, b]$  follows shortly via Heine-Borel discussions; Heine-Borel result is also needed here to prove that compact set in  $\mathbb{R}$  implies and implied by closed and bounded set; discussions on connectedness will appear shortly in due course].

MS having Bolzano- Weierstrass property (BWP) and sequentially compact MS. Results:

- (a)  $(M, \rho)$  is sequentially compact MS is equivalent to  $(M, \rho)$  having BWP.

(b) Compact MS implies sequentially compact MS. **(8)**

Total Boundedness and proofs of four equivalent statements:

- (a)  $(M, \rho)$  is compact MS
- (b)  $(M, \rho)$  is sequentially compact MS
- (c)  $(M, \rho)$  is complete MS and totally bounded
- (d)  $(M, \rho)$  has BWP.

Statement of Fixed Point Theorem with illustrations of its applications.

Heine-Borel and Bolzano-Weierstrass Theorem of RNS:

- (a) Heine-Borel Theorem:  $A$  is a closed and bounded subset of  $\mathbb{R}$  implies  $A$  is compact.
- (b) Bolzano-Weierstrass Theorem: Bounded infinite subset of  $\mathbb{R}$  must have limit point.

Separated and connected sets in a MS: Result:

$E(\subseteq \mathbb{R})$  is a connected set iff  $x \in E, y \in E, x < z < y$  implies  $z \in E$ . **(8)**

**Part II**  $(M, \rho)$  with special reference to  $(\mathbb{R}, d)$

Result:  $(p_n \rightarrow p)$  implies  $p$  is unique.

For sequences in  $(\mathbb{R}, d)$ , we can study the relation between convergence and algebraic operations, e.g.  $\lim(x_n + y_n) = \lim x_n + \lim y_n$  etc.

Results:

- (a) In a compact MS, each sequence has a convergent subsequence.
- (b) Bounded sequence in  $\mathbb{R}$  must have a convergent subsequence.

Results: Subsequential limits form a closed set.

Link:  $\lim \sup / \lim \inf$  for bounded sequence in  $\mathbb{R}$ . Details in Analysis II. (Closed and bounded sets i.e. compact sets in  $\mathbb{R}$  must have maximum/minimum elements. Thus the set of subsequential limits must have maximum/minimum elements etc.)

Results:

- (a) If  $(M, \rho)$  is compact MS, then a Cauchy sequence in it must converge.
- (b) In  $(\mathbb{R}, d)$ , each Cauchy sequence must converge.

Note:  $(\mathbb{R}, d)$  is complete in Cauchy sequence sense too. **(5)**

In  $(\mathbb{R}, d)$ , we can consider monotone increasing / monotone decreasing sequences—details in Analysis II.

$f: (MS)_1 \rightarrow (MS)_2$

Limit and continuity for such functions.

$(X, d_X)$  and  $(Y, d_Y)$  are Metric Spaces.  $E \subseteq X, f: E \rightarrow Y$ .

(Special interest:  $X = Y = \mathbb{R}, d_X = d_Y = d$ )

Function limit is a sequence limit. Limit is unique. For the case  $(\mathbb{R}, d)$ :  $\lim_{x \rightarrow p} (f + g) = A + B$  etc.

Sandwich Rule and Cauchy condition for existence of finite limit.

Continuous functions:  $d_Y(f(x), f(p)) < \varepsilon \leftarrow x \in E, d_X(x, p) < \delta$ .

Continuity / limit: link—if  $p$  is a limit point of the domain of  $f$ .

Continuity of composite functions.

With respect to real MS:  $f+g, f-g, f \cdot g$  etc. are continuous.

For continuity with respect to MS, we may consider  $f: MS \rightarrow MS, f$  continuous, instead of  $E \rightarrow MS$  etc.

Continuity and compactness in  $(\mathbb{R}, d)$ :

Range( $f$ ) is bounded and bounds are attained for continuous  $f: [a, b] \rightarrow \mathbb{R}$ .

Continuity of inverse functions with respect to injective functions on compact MS.

Results related to continuity / injectivity/ monotonicity/ invertibility in  $(\mathbb{R},d)$ :

- (a) Let  $f:[a,b] \rightarrow \mathbb{R}$  be continuous and injective. Then  $f$  is strictly monotonic function.
- (b) Let  $f:[a,b] \rightarrow \mathbb{R}$  be continuous and injective. Then  $f^{-1}$  is continuous. (This is linked to invertibility of injective/ continuous function on compact set.)
- (c) Let  $f:[a,b] \rightarrow \mathbb{R}$  be continuous and strictly monotone increasing with  $c = f(a)$ ,  $d = f(b)$ . Then (1)  $f([a,b]) = [c,d]$ , (2)  $f^{-1}$  is strictly monotone increasing on  $[c,d]$ , (3)  $f^{-1}$  is continuous (same results for strictly monotone decreasing function).
- (d) Let  $f:[a,b] \rightarrow \mathbb{R}$  be continuous and let it assume every value between (inclusive)  $f(a)$ ,  $f(b)$  exactly once in  $[a,b]$ . Then  $f$  is strictly monotone on  $[a,b]$ .

Uniform Continuity: Results—

$f:X \rightarrow Y$  is continuous,  $X$  compact, implies  $f$  is uniformly continuous ( $X,Y$  are MS).

Lipschitz's condition and uniform continuity for real functions of real variable.

Results:

- (a) Let  $f$  be continuous on  $(a,b)$ . Necessary and sufficient condition for  $f$  to be uniformly continuous on  $(a,b)$  is that  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow b^-} f(x)$  both exist finitely.
- (b) Let  $f$  be continuous on  $(a,b)$ . Then  $f$  admits of continuous extension on  $\mathbb{R}$  iff  $f$  is uniformly continuous on  $(a,b)$ .

Continuity and connectedness in Real MS:

Let  $f:X \rightarrow Y$  is continuous ( $X,Y$  are MS) and  $E$  is connected subset of  $X$ . Then  $f(E)$  is connected.

Result:  $f$  is continuous real function on  $[a,b]$ . Let  $f(a) < c < f(b)$  and  $f(a) < c < f(b)$ . Then there exists  $x \in (a,b)$  with  $f(x) = c$ . (Intermediate Value Property)

Discontinuities with respect to real functions on  $(a,b)$  with special reference to discontinuities of monotone functions.

Right Hand Limit and Left Hand Limit of  $f$  at  $x$  ( $f$  is defined on  $(a,b)$ )—Discontinuities of first/second kind. Discontinuities of monotone functions.

Different types of limits for real functions of real variables. Definition of  $\lim_{x \rightarrow \infty} f(x) = A$ ,  $\lim_{x \rightarrow \infty} f(x) = \infty$ ,  $\lim_{x \rightarrow c} f(x) = \infty$ . (15)

Differentiation/ Mean Value Theorems etc.

Real functions on intervals. Definition of  $f'(c)$ —Result:

Differentiability implies continuity; converse does not hold.  $(f+g)'$ ,  $(f.g)'$  etc.

Meaning of sign of derivative. Chain Rule—differentiation of function of function. Differentiation of inverse functions.

Rolle's Theorem, Lagrange's Theorem—Monotonicity of functions.

Result:  $f$  is continuous at  $x = c$  and  $\lim_{x \rightarrow c} f'(x) = L$  implies  $L = f'(c)$ .

Corollary: Let  $f'$  exist in  $(a,b)$ ; then  $f'$  cannot have discontinuity of first kind in  $(a,b)$ .

Darboux Theorem for  $f'$  in  $[a,b]$ . Corollary:  $f'$  cannot have discontinuity of first kind in  $[a,b]$ .

Lipschitz condition is equivalent to boundedness of derivative.

Cauchy Mean Value Theorem.(10)

Derivatives of higher order. Leibnitz's Theorem.

Taylor/ Maclaurin Series (finite form). Taylor/ Maclaurin Series (infinite form).

Function expansion for:  $\sin x$ ,  $e^x$ ,  $\ln(1+x)$ ,  $(1+x)^m$  etc.

(Elementary notions of infinite series of real numbers – statements only—details in Analysis II)(8)

Young's form of Taylor Formula.

Maxima/ Minima of differentiable functions – necessary conditions—general form of sufficient conditions via Taylor Finite Expansions.

Maxima/ Minima of non- differentiable functions via sign change of  $f'$ -- local/global extremum.

Indeterminate Form of function limits—Proof of L’Hospital’s Rule via Cauchy MVT—This rule should be used only when standard limits fail—otherwise circularity of argument may get involved.(4)

**Total No. of classes for general teaching:90. No. of tutorial classes:45.**

- Books Recommended:** (1) Mathematical Analysis—W.Rudin  
(2) Mathematical Analysis—T.M.Apostol  
(3) A Course of Mathematical Analysis(Vol. I&II)—S.M.Nicholsky  
(4)Mathematical Analysis- Shanti Narayan  
(5) Introduction to Real Variable Theory—S.C.Saxena,S.N.Shah  
(6) Metric Space—E.T.Copson  
(7) Introduction to Topology and Modern Analysis—G.F.Simmons  
(8) Advanced Calculus—Schaum Series  
(9) Real Analysis—R.Goldberg

**Paper III, Module2** (Algebra II)

**50 marks**

Normal subgroups and Quotient Groups(4). Homomorphism and Isomorphism of Groups, Cayley’s Theorem, Isomorphism Theorems(8).

Rings, Integral Domains, Division Rings, Fields. Subrings and subfields(10).Characteristic of a Ring(5). Ideals, Quotient Rings and Ring Homomorphisms, Principal Ideal Domains(PIDs), Isomorphism Theorems(12). Congruence Arithmetic—Fermat’s and Wilson’s Theorem(3), Chinese Remainder Theorem(2).Prime Ideals and Maximal Ideals(5). Embeddings of rings, Every Integral Domain is embeddable in a field(6).

Factorization in Integral Domains,Fatorisation Domain, NASC for anIntegral Domain to be a FD Unique Factorization Domain(UFD), every PID is a UFD(15), Euclidean Domain(ED), every ED is a PID and every ED is a UFD,Every PID is a UFD Euclidean Algorithm in ED, in particular in  $\mathbb{Z}$  and  $\mathbb{Z}[i]$ .(10).GCD-Relevant Theorems.

**Total No. of classes for general teaching:90. No. of tutorial classes:45.**

- Books Recommended** (1) Algebra—M.Artin  
(2)Topics in Abstract Algebra—M.K.Sen,S.Ghosh, P.Mukhopadhyay  
(3) Topics in Algebra—I.N.Herstein  
(4) Modern Algebra—S.Singh, Q.Zameruddin  
(5) First Course in Abstract Algebra—J.B.Fraleigh  
(6) Abstract Algebra—D.S.Dummit, R.M.Foote  
(7) Modern Algebra with Applications—W.J.Gilbert  
(8) Higher Algebra(Abtract and Linear)—S.K.Mapa

**PAPER-IV**

**Paper IV, Module1** (Linear Programming and Boolean Algebra)

**50 marks**

Linear Programming

Linear Programming and its assumptions(1), standard and canonical form(2), examples of Linear Programming Problems(3), Geometric Solution(2).

Linear Algebra, Convex Analysis and Polyhedral Sets; Euclidean Space(1), linear dependence and independence, spanning set and basis(2), Replacing a vector in a basis by another vector(1).

Revision of Matrices: elementary row operations and its use in solving a system of linear equations and in finding rank of a matrix(2), simultaneous linear equations ( both homogeneous and non-homogeneous), Theorem on consistency of a system of linear equations(3).

Definition and examples of convex sets(2), Extreme points, Hyperplane and Halfspaces, directions and extreme directions of a convex set(2), Representation Theorem of polyhedral sets—bounded and unbounded (statement only)(2).

The Simplex Method: Extreme points and Optimality, definition of basic feasible solution (b.f.s.), correspondence between b.f.s. and extreme points(4), geometric motivation of the simplex method, algebra of the simplex method, interpretation of entering and leaving the basis, unboundedness(8).

The Simplex Algorithm—its finite convergence in the absence of degeneracy(2), the simplex method in tableau format, working out of sums(5).

Starting solution and convergence: obtaining initial b.f.s.—introduction of artificial variables(1).

Eliminating artificial variables—the Two-Phase Method(3), The Charnes' Big-M Method(2). Degeneracy, Cycling and Stalling(3).

Duality in LP- Formulation of the Dual, Dual of the dual is primal(3), Primal-dual relationships—the Fundamental Theorem of Duality, Complementary Slackness(6).

Transportation and Assignment Problems(10). Introduction to Game Theory(10).

### **Boolean Algebra**

Posets—definition and examples. LUB and GLB. Lattice—some elementary results. Duality Principles. Lattice Homomorphisms. Direct product of lattices. Distributive lattice—Boolean Algebra as a complemented distributive lattice. Examples—some basic theorems. Boolean Polynomial and Functions. Representation Theorem for finite Boolean Algebras —statement and examples. (5). Introduction to switching circuit and its application to Boolean algebra.

**Total No. of classes for general teaching:100. No. of tutorial classes:35.**

**Books Recommended:** (1) Linear Programming and Network Flows-- M.S.Bazaraa, J.L.Jarvis, H.D.Sherali  
(2) Modern Algebra with Applications—W.J.Gilbert

### **Paper IV, Module 2** (Mechanics I)

**50 marks**

**[ Theoretical derivation will be done preferably either by vector method or by matrix method ]**

Fundamental Principles of Dynamics(2); Laws of Motion(1); Motion in a straight line under variable acceleration(5). Motion of a particle tied to an elastic string or elastic spring(2). Motion of a connected system(2).

Different co-ordinate systems for describing motion of a particle in a plane and expressions for velocity and acceleration of such a particle in these systems(4)—Cartesian co-ordinate system and motion of a particle in a plane under different laws of resistance. Motion of a projectile in a resisting medium(5).

Polar co-ordinate system(4), Rotating co-ordinate system and coriolis force(3). Central orbits and stability of nearly circular orbits(10). Inverse square law and classification of orbits(3).

Tangent normal co-ordinate system and constrained motion(6). Work, power, energy, conservative force field(2), Principles of conservation of energy and linear as well as angular momentum(3). Motion of a particle of varying mass such as raindrops and launched rockets(3).

Two possible motion of a rigid body—translation and rotation(2). Kinetic energy of a rigid body and Inertia Matrix(1). Diagonalisability of the Inertia Matrix and emergence of the ideas of principal moments and principal axes(2). Momental ellipsoids(1). Determination of moments of inertia of various rigid bodies(3)—Theorems of parallel axes and perpendicular axes(1). Equipomental bodies and correlated problems(3).

D'Alembert's Principle and general equations of motion of a rigid body(4). Independence of motion of centre of inertia and motion relative to the centre of inertia(2). Motion of a rigid body about a fixed axis(5). Compound Pendulum(4).

Motion of a rigid body in two dimensions(2). Motion of a solid sphere down an inclined plane—various possible subcases(3). Motion of a circular disc(4). Principles of conservation of linear momentum, angular momentum and energy under finite forces(3).

**Total No. of classes for general teaching:100. No. of tutorial classes:35.**

**Books Recommended:** (1) Classical Mechanics—N.C.Rana, P.S.Joag  
(2) Rigid Dynamics—Md. Motiur Rahaman  
(3) Dynamics of a Particle and of Rigid Bodies—S.L.Loney  
(4) Statics—A.S.Ramsey

### THIRD YEAR

#### PAPER-V

**Paper V, Module 1** (Probability Theory)

**40 marks**

Random Experiments(2). Classical definition of Probability and its limitations(2). Kolmogorov's axiomatic development through sigma-algebra approach(4). Conditional probability and Bayes' rule(6), Independence of events—simple problems without combinatorial complications(2).

Compound experiments and independent trials(2). Bernoulli and Poisson trials(2), Binomial law and its Poisson approximation(2), simple problems(2).

Random variables and induced probability measure(2). Distribution function and their properties(2). Concept of p.m.f. and p.d.f.(1). Examples of discrete and continuous distribution in one dimension(2). Transformation of one dimensional random variables—simple problems(4). Mathematical Expectation of random variables and its properties(2). Raw and central moments—special stress on mean, variance, skewness and kurtosis of a distribution(4). Mode and median; a comparative study of mean, median and mode(2).

Determination of moments by moment-generating function and characteristic function(1). Computation of characteristic function of important one-dimensional distributions(2).

Two-dimensional probability distributions—uniform and bivariate Normal distributions(2). Conditional distributions(1). Transformation of two dimensional random variables – simple problems(3). Two dimensional expectations(2), multiplication rule for expectations(1). Conditional expectation and regression(1). Quadratic and linear regression curves by least square method(2). Independence of random variables(2), Correlation coefficient(1). Joint characteristic function(2).

Chebyshev's inequality and related simple problems(2). Jensen's inequality(1). Idea of convergence in probability and convergence in distribution(4). Laws of large numbers(1). Statement of Uniqueness Theorem and Continuity Theorem for characteristic functions, Poisson and Normal

approximation of Binomial distribution(4). Statement of Central Limit Theorem for equal component case—Lindeberg's and Liapounoff's criteria(2).

**Total No. of classes: 90. No tutorial class.**

Book Recommended: (1) The elements of Probability Theory and some of its applications—

H. Cramer

(2) An Introduction to Probability Theory and its Applications (Vol. I)—

W. Feller

(3) Theory of Probability—B.V.Gnedenko

(4) Mathematical Probability—J.V.Uspensky

(5) Introduction to Probability and Mathematical Statistics—A. Gupta

(6) Introduction to Probability and Statistics—V.K.Rohatgi

(7) Probability Models—Sheldon E. Ross

(8) Introduction to Probability—Hoel, Port, Stone

**PaperV, Module 2** (Numerical Analysis Theory and Practical) **30+30=60 marks**

**Theory:**

**30 marks**

What is Numerical Analysis? (1)

Errors in Numerical Computations : Machine Epsilon error , Relative error , Absolute error , Percentage error , round-off rules and Round-off error ,inherent errors , Significant digits and Numerical instability.

Error of a sum, difference, product and quotient of two approximate numbers (4).

Operators :  $\Delta, \nabla, \mu, \delta, E$  (Definitions and simple relations among them ) (2).

Interpolation and discrete approximation: Problems of interpolation, Weierstrass' approximation theorem (statement only ).Polynomial interpolation and Vandermonde's determinant. Equispaced arguments. Difference Table. Deduction of Newton's Forward and Backward interpolation formulae. Statements of Stirling's and Bessel's interpolation formulae. Error terms. General Interpolation Formulae: Deduction of Lagrange's interpolation formula and its error estimate.Drawback of Lagrangian Interpolation Formula and preference for divided difference. Newton's General Interpolation Formula (statement only). Inverse interpolation. Spline Interpolation Technique. (11)

Interpolation formulae using the values of both  $f(x)$  and its derivative  $f'(x)$  : Idea of Hermite interpolation formula (basic concepts only).(3)

Numerical Differentiation based on Newton's Forward and Backward and Lagrange's formula(4).

Numerical Integration: Integration of Newton's interpolation formula. Newton- Cote's formula. Basic Trapezoidal and Simpson's 1/3 rd formulae. Their composite forms. Weddle's rule (statement only). Romberg integration, Double integration, methods based on Undetermined Co-efficients. Statement of the error terms associated with these formulae. Degree of precision (definition only)(8)

Numerical Solution of non-linear equations: Location of a real root by tabular method. Bisection method. Secant/ Regula-Falsiand Newton-Raphson methods, their geometrical significance. Fixed point iteration method..(7)

Numerical solution of a system of linear equations : Gauss elimination method: idea of contraction map and Lipschitz's constant.Gauss- Jordan elimination method. Iterative method—Jacobi iteration method, Gauss- Seidal method. Matrix inversion by Gauss elimination method (only problems—up to 3x3 order).(5)

Vector iteration and eigenvalue problems: Power method for numerically extreme eigenvalue (3)  
 Numerical solution of Ordinary Differential Equation: Basic ideas, nature of the problem, LU factorization and pivoting. Existence and uniqueness of solution. Picard, Euler and Runge- Kutta (4th order) methods (emphasis on the problems only) (6).

**Practicals:**

**30 marks**

Newton's Forward and Backward Interpolation. Stirling and Bessel Interpolation. Lagrange's and Newton's Divided Difference interpolation . Inverse interpolation.

Numerical integration: Trapezoidal and Simpson 1/3<sup>rd</sup> rule.

Numerical solution of non-linear equations: Tabulation, Bisection, Secant/ Regula Falsi, Fixed point iteration method, Newton-Raphson method.

Solution of system of Linear equation: Gauss elimination Gauss- Jordan elimination method Jacobi iteration method, Gauss- Seidal method.

Eigenvalue Problems-Numerically largest eigenvalue by power method.

Solution of Differential equation :Runge Kutta Method (4<sup>th</sup> Order)

Use of computer package.

*(Above problems are to be done on a non-programmable scientific calculator)*

**Total No. of General Classes: 60. No. of practical classes:60.**

**Books Recommended** (1) Numerical Analysis—J. Scarborough

(2) Introduction to Numerical Analysis—A. Gupta, S.C. Bose

(3) Numerical Methods—W. Boehm, H. Prantzs

(4) Numerical Methods for Scientific and Engineering Computation (4th Ed.)  
 -- Jain, Iyengar, Jain

(5) Computational Mathematics—B.P. Demidovich, I.A. Maron

(6) Numerical Analysis—Ralston, Rabinowitz

**Paper VI , Module I** (Analysis II)

**50 marks**

Sequence and series of Real Numbers

Sequences—bound, limit, convergence, non-convergence, limit operations, Sandwich rule, monotone sequence. Convergence results regarding bounded, monotone sequence. NIP of R-Cauchy sequence (CS)- CS convergence in R. Limit of some important particular sequences like  $x_n =$

$$\left(1 + \frac{1}{n}\right)^n \text{ etc.}$$

Null sequences—results and applications. Cauchy's First and Second Limit Theorems. Subsequences. Bounded sequence must have convergent subsequences. Lim sup, Lim inf etc. (Details needed only for those parts which are not covered earlier in Analysis-I).(10)

Infinite series of real numbers—convergence/divergence—Cauchy condition for convergence.

Series of positive real numbers—Comparison, Ratio, Root Test, Cauchy Condensation Test, Raabe's Test. Statement / application only for: Kummer Test, Bertrand's Test, Logarithmic, Gauss Test.

Absolute and conditional convergence—Alternating series—Leibnitz's Test. Statement/ application only for Abel's , Dirichlet's Test.

Insertion/ removal of brackets in Infinite Series.

Statement of Riemann Rearrangement Theorem.

Euler constant  $\gamma$  and re-arrangement of alternating harmonic series. Abel- Pringsheim Result(12).

### Sequence and series of real-valued functions of real variables

Function Sequence—pointwise and uniform convergent—Cauchy condition for uniform convergence. Limit Function—boundedness, repeated limits, continuity, differentiability, integrability—under uniform convergence. Dini's Theorem.(8)

Function Series—pointwise/uniform convergence. Cauchy condition for uniform convergence.

Passage to the limit term by term. Boundedness, continuity of sum function under uniform convergence and term by term boundedness/ continuity of the given series.

Term by term differentiation/ integration of series under uniform convergence. Tests for uniform convergence: Weierstrass's M-test, Abel, Dirichlet's Test.(8)

Power Series Cauchy- Hadamard Theorem, determination of radius of convergence, Uniform and Absolute convergence.

Properties of sum function. Abel's Limit Theorem. Uniqueness of power series having same sum function.  $E^x$ ,  $\ln(1+x)$ ,  $\sin x$ ,  $(1+x)^m$  etc.defined as power series and deduction of their salient properties.(7)

Functions (real) of several (real) variables Point sets in  $R^2, R^3$  and basic topology in  $(R^2, d)$  and  $(R^3, d)$  as already considered under Metric Space in Analysis I.

Passing mention to  $(R^n, d)$  ( $d$ —Euclidean metric).

For functions  $R^2 \rightarrow R$ ,  $R^3 \rightarrow R$ —limit, continuity, partial derivative. Sufficient condition for continuity. Discussions on repeated limits and double limits.

For functions  $R^2 \rightarrow R$ —differentiability and its sufficient condition.

Differential as a map. Chain Rule, Euler Theorem and its converse. Schwarz's and Young's Theorem.(8)

Jacobian for real functions of two/three variables. Properties of Jacobians. Functional dependence.

Statement/ applications of Implicit Function Theorem (two/three variables).Differentiation related to Implicit Function Theorem. Jacobians related to implicit functions. Partial derivatives as ratio of Jacobians in case of two variable functions.(8)

MVT and Taylor Formula (finite/infinite) for  $f: R^2 \rightarrow R$ . Function extreme for 2/3 variables real functions. Lagrange's method of undetermined multipliers.(5)

Riemann Integration For Bounded Functions Partition and Refinement of an interval.Upper Darboux Sum  $U(P, f)$  and lower Darboux Sum  $L(P, f)$  and associated results. Upper Riemann Integral and Lower Riemann Integral. Darboux's Theorem. Necessary and sufficient condition of R-integrability(2).Classes of R-integrable functions: monotone function, continuous function, piecewise continuous functions with (1) finite number of points of discontinuities, (2) infinite number points of discontinuities having finite number of accumulation points.(2)

Riemann Sum: alternative definition of integrability. Equivalence of the two definitions(statement only) (1). Integrability of sum, product, quotient, modulus of R-integrable functions(2). Sufficient condition for integrability of composition of R-integrable function. Properties of R-integrable functions arising from the above results.

Function defined by definite integral  $\int_a^x f(t) dt$  and its properties. primitive or indefinite

integral(1).Properties of definite integral. Definition of  $\log x$  ( $x > 0$ ) as an integral and deduction of simple properties including its range(2). Definition of  $e$  and its simple properties(1). Fundamental Theorem Of Integral Calculus. First Mean Value Theorem of Integral Calculus. Statements and applications of Second Mean Value Theorem of Integral Calculus (both Bonnet's and Weierstrass' form)(2). Theorem on method of substitution for continuous function(1).

### Improper Integral

Range of integration, finite or infinite. Necessary and sufficient condition for convergence of Improper Integral in both cases (1).

Tests of convergence : Comparison and  $\mu$ -test. Absolute and non-absolute convergence—corresponding tests(2). Beta and Gamma functions—their convergence and inter-relations(2).

Statement of Abel's and Dirichlet's Tests for convergence of the integral of a product(1).

Uniform convergence of Improper Integral by M-test

$$\left[ \Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}, 0 < n < 1, \text{to be assumed} \right] \text{(1). Problems(2).}$$

Definite Integral as a function of a parameter : Differentiation and Integration with respect to the parameter under integral sign—statements (only) of some relevant theorems and simple problems(4).

Fourier Series : Trigonometric Series. Fourier co-efficients. A periodic function of bounded variation can be expressed as a Fourier Series (statement only) (2). Statement of Dirichlet's conditions of convergence (1). Half-range series, sine and cosine series(1). Problems(2).

Double Integral Concept of Upper Sum, Lower Sum, Upper Integral, Lower Integral and Double Integral (no rigorous statement is needed) (1). Statement of Existence Theorem for continuous functions. Change of order of integration(2). Triple integral. Transformation of double and triple integrals (problems only) (2). Determination of volume and surface area by Multiple Integrals (problems only) (1).

**Total No. of classes for general teaching:105. No tutorial class.**

- Books Recommended:** (1) Mathematical Analysis—T.M.Apostol  
(2) Principles of Mathematical Analysis—W. Rudin  
(3) Mathematical Analysis—Shanti Narayan  
(4) Introduction to Mathematical Analysis—A. Gupta  
(5) Advanced Calculus—Schaum Series  
(6) Introduction to Real Variable Theory—S.C.Saxena,S.M.Shah  
(7) Methods of Real Analysis—R.Goldberg  
(8) Calculus (Vol. I,II)—Courant and John  
(9) Advanced Calculus—D. Widder

### Paper VI , Module 2 (Optional 1)

**50 marks**

Any **one** from the topics (1), (2), (3) and (4) below :

#### **(1) Mechanics 2**

Impact of elastic bodies and Newton's experimental law of impact(1). Direct and oblique impact of elastic spheres(2) . Loss of K.E. in both cases. Angle of deflection(2).

Equations of motion under impulsive forces(2). Equations of motion of a rigid body about a fixed axis under impulsive forces and centre of percussion(2). Principles of conservation of linear and angular momentum under impulsive forces. Work done by impulsive forces(1). Impulsive forces acting on a rigid body moving in 2D(7).

Analytical reduction of a given system of forces acting on (1) a particle, (2) a rigid body(3)..Astatic equilibrium of a system of coplanar forces acting at different points of a body and Astatic

centre(2). Detailed discussion on principle of virtual work(10). Conditions of equilibrium of rigid bodies in 2D and 3D (1). Converse of principle of virtual work(3).

Forces in 3D—Poinsot's central axis and its uniqueness(3). Wrench, Pitch, Intensity, Invariant of a given system of forces(4).

Types of equilibria and the potential at equilibrium—static and dynamic equilibrium. Metastable, stable and unstable equilibrium(2). Conservative force field(1). Energy test of stability due to Lagrange(5). Hooke's law as a consequence of stable equilibrium(1). Stability of central orbits from the viewpoint of effective potential energy(1). Use of energy test of stability to check whether the equilibrium state of a perfectly rough heavy body resting on a fixed body is stable or not(2).

Definition of fluid and perfect fluid. Transmissibility of fluid pressure. Pascal's Law. Elementary results on pressure of heavy fluids and thrusts on plane surfaces. Centre of pressure (c.p.): formula for depths of c.p. of a plane area—e.g. triangular, circular and elliptic area. Position of the c.p. referred to co-ordinate axis through the c.g. of the plane area. Pressure derivative in terms of force. Surface of equipressure. Determination of necessary and sufficient condition of equilibrium of a fluid under action of a force field. Conservative force field and equipotential surface. Stability of equilibrium of floating bodies. Metacentre, Plane of flotation, surface of buoyancy and derivation of condition of stability.(30)

**Total No. of classes: 105. No tutorial class.**

Book Recommended: (1) Dynamics of a Particle and of Rigid Bodies—S.L.Loney

(2) Analytical Statics—M.C.Ghosh

(3) Hydrostatics—A.S.Ramsay, W.H.Besant

(4) Hydrostatics—J.M.Kar

(5) Statics—F.D.Chorlton

## (2) Topology

Definition and examples of Topological Space, neighbourhood system, interior point, limit point, interior, closure, open and closed sets, base of a topology.

Continuous functions—definition and examples, equivalent criteria for continuity, homeomorphism.

Revising metric space, Tietze extension theorem.

Compactness—definition and examples, closed and bounded subsets of  $E_n$  are compact and vice versa, compactness is a topological property, closed subset of a compact space is compact, compact subset of a Hausdorff space is closed, an infinite subset of a compact space must have a limit point, continuous real-valued function defined on a compact space is bounded and attains its bounds, Lebesgue's lemma on compact metric space.

Product space of finite number of topological spaces—definition and examples, product topology is the smallest topology for which all the projections are continuous,  $f: Z \rightarrow X \times Y$  is continuous iff composite functions  $p_1 f: Z \rightarrow X, p_2 f: Z \rightarrow Y$  are both continuous,  $X \times Y$  is Hausdorff iff both  $X$  and  $Y$  are Hausdorff,  $X \times Y$  is compact iff both  $X$  and  $Y$  are compact, a subset of  $E_n$  is compact iff it is closed and bounded, Tychonoff theorem (statement only).

Connectedness – definition and examples, a nonempty subset of the real line is connected iff it is an interval, equivalent criteria for connectedness, continuous image of a connected space is connected, connectedness is a topological property, a space is connected if it contains a dense

connected subset,  $X$  and  $Y$  connected implies  $X \times Y$  connected. Path connectedness—definition and examples, a path connected space is connected, a connected open subset of a Euclidean space is path connected.

The Quotient Space.

Homotopy Theory—an introduction.

**Total No. of classes: 105. No tutorial class.**

Book Recommended: 1. Basic Topology—M.A. Armstrong (Springer)

2. Topology---A First Course—James Munkers

### (3) Measure Theory and Integration

Extended Real Number System  $\mathbb{R}^*$ .

The Length function  $l$ : its monotonicity, finite additivity and countable subadditivity.

Extending the length function  $l$ : General Extension Theory

First extension of the length function  $l$  from semi-algebra of intervals to the algebra of set of all finite unions of disjoint intervals. Definition of set function, semi-algebra and algebra of sets. General discussions of extending a set function from a semi-algebra to the algebra generated by the semi-algebra. Impossibility of extending the length function  $l$  to all subsets of  $\mathbb{R}$ —Ulam's Theorem (statement only). Problems of countably additive set functions  $\mu$  on an algebra  $\mathcal{A}$ :  $\mu(A-B) = \mu(B) - \mu(A)$ ,  $A \subseteq B$ ,  $A, B \in \mathcal{A}$ ;  $\mu(\Phi) = 0$ ;  $\mu$  is finitely additive implies  $\mu$  is monotone; if  $\mu(\Phi) = 0$  then [ $\mu$  is countably additive iff  $\mu$  is both finitely additive and countably subadditive].

Approximating the length of a set  $A \subseteq \mathbb{R}$ : The Induced Measure

Definition of outer measure  $\mu^* : P(X) \rightarrow [0, \infty]$  induced by a measure  $\mu : \mathcal{A} \rightarrow [0, \infty]$  ( $\mathcal{A}$  is an algebra of subsets of  $X$ ).

Properties of outer measure—monotonic, countable subadditive extension of  $\mu$ . Definition of  $\mu^*$ -measurable sets—equivalent criteria. Definition and examples of  $\sigma$ -algebra.  $\sigma$ -algebra of Borel subsets of a Topological Space  $X$ .  $\sigma$ -algebra  $S(C)$  generated by a collection  $C$  of subsets of a non-empty set  $X$ . Properties of the collection  $S^*$  of  $\mu^*$ -measurable sets--  $\mathcal{A} \subseteq S^*$ ;  $\mu^*/S^*$  is countably additive;  $S^*$  is a  $\sigma$ -algebra. Equivalent ways of describing  $\mu^*(E)$ ,  $\mu^*$  is the outer measure induced by a measure  $\mu : \mathcal{A} \rightarrow [0, \infty]$ ,  $\mathcal{A}$  an algebra of subsets of  $X$ ,  $E \subseteq X$ .

Results: (a) For every  $E \subseteq X$ , there exists a set  $F \in S(\mathcal{A})$  such that  $E \subseteq F$ ,  $\mu^*(E) = \mu^*(F)$  and  $\mu^*(E - F) = 0$ , (b) Let  $\mu$  be a measure on an algebra  $\mathcal{A}$  of subsets of a set  $X$  and let  $\mu^*$  be the induced outer measure. Let  $E \in S^*$  be such that  $\mu^*(E) < \infty$  and let  $\varepsilon > 0$  be arbitrary. Then there exists a set  $F \in \mathcal{A}$  such that  $\mu^*(E \Delta F) < \varepsilon$ .

The Lebesgue Measure on  $\mathbb{R}$  and Its Properties

The unique extension of the length function—the Lebesgue measure: Lebesgue outer measure  $\lambda^*$ , Set  $L$  of Lebesgue measurable sets, Lebesgue measure space and Lebesgue measure. Properties of  $\lambda^*$ . Relation of Lebesgue measurable sets with topologically nice subsets of  $\mathbb{R}$ :

(a) Let  $E \subseteq \mathbb{R}$  and  $\lambda^*(E) < \infty$ . Then given  $\varepsilon > 0$ , there exists a set  $F_\varepsilon$  which is a finite disjoint union of open intervals and is such that  $\lambda^*(E \Delta F_\varepsilon) < \varepsilon$ .

(b) For any set  $E \subseteq \mathbb{R}$ , the following statements are equivalent:

- (1) E is Lebesgue measurable,
- (2) For every  $\varepsilon > 0$ , there exists a closed set  $F_\varepsilon$  such that  $F_\varepsilon \subseteq E$  and  $\lambda^*(E - F_\varepsilon) < \varepsilon$ ,
- (3) For every  $\varepsilon > 0$ , there exists An open set  $G_\varepsilon$  such that  $E \subseteq G_\varepsilon$  and  $\lambda^*(G_\varepsilon - E) < \varepsilon$
- (4) There exists a  $G_\delta$  set G such that  $E \subseteq G$  and  $\lambda^*(G - E) = 0$ ,
- (5) There exists an  $F_\sigma$  set F such that  $F \subseteq E$  and  $\lambda^*(E - F) = 0$ .

(c) Let E be Lebesgue measurable with  $0 < \lambda(E) < \infty$  and let  $\varepsilon > 0$ . Then there exists a compact set  $K \subseteq E$  such that  $\lambda(E - K) < \varepsilon$ .

Properties of the Lebesgue measure with respect to the group structure on R:

- (a) Let  $E \in L$ . Then  $E + x \in L$  for every  $x \in R$  and  $\lambda(E + x) = \lambda(E)$ .
- (b) Let  $E \in L$  and  $x \in R$ . Let  $X_e = \{xy / y \in E\}$ . Then  $xE \in L$ , for every  $x \in E$ .

Example (due to Vitali) of nonmeasurable subset of R.

### Integration

Let  $(X, S, \mu)$  be a fixed measure space.

Set  $L_0^+$  of nonnegative simple functions  $s: X \rightarrow R^*$ . Integral of nonnegative simple functions: definition and properties. If  $s \in L_0^+$  and  $\{s_n\}$  is any increasing sequence in  $L_0^+$  such that  $\lim_{n \rightarrow \infty} s_n(x) = s(x), x \in X$ , then  $\int s d\mu = \lim_{n \rightarrow \infty} \int s_n d\mu$  and  $\int s d\mu = \sup \{ \int s' d\mu / 0 \leq s' \leq s, s' \in L_0^+ \}$

### Extending the integral beyond nonnegative simple function

$L^+$ , the class of all those functions  $f: X \rightarrow R^*$  for which there exists an increasing sequence of functions  $\{s_n\}$  from  $L_0^+$  such that  $f(x) = \lim_{n \rightarrow \infty} s_n(x)$ , for all  $x \in X$ . Definition of  $\int f d\mu$ , where  $f \in L^+$ .  $f \in L^+$  iff there exist functions  $s_n \in L_0^+$  such that  $0 \leq s_n \leq f$  for all n and  $f(x) = \lim_{n \rightarrow \infty} s_n(x)$ , for all  $x \in X$ . Properties of  $\int f d\mu, f \in L^+$ . Monotone Convergence Theorem. Intrinsic characterization of  $L^+$ -- equivalent criteria for  $f: X \rightarrow R^*$  to belong to  $L^+$ . Definition of S-measurable function:  $f: X \rightarrow R^*$  is S-measurable iff both  $f^+$  and  $f^- \in L^+$ . Equivalent criteria of S-measurability. If  $f_n: X \rightarrow R^*$  be measurable ( $n = 1, 2, \dots$ ), then  $\sup_n f_n, \inf_n f_n$  are measurable ; in particular, if  $\{f_n\}$  converges to f, then f is measurable.

Fatou's Lemma.

Integrable Functions: Definition of  $\mu$ -integrable functions and  $\mu$ -integral.  $L_1(X, S, \mu)$ —the space of all integrable functions on X. Properties of  $L_1(X, S, \mu)$  and of  $\int f d\mu$ . Lebesgue's Dominated Convergence Theorem.

### The Lebesgue Integral and its relation with the Riemann Integral

The Special Case: Set  $L_1(R, L, \lambda)$  of Lebesgue integrable functions where R is the set of real numbers, L is the  $\sigma$ -algebra of Lebesgue measurable sets and  $\lambda$  is the Lebesgue measure.

Notation: For any set  $E \in L$ ,  $L_1(E)$  stands for the space of integrable functions on the measure space  $(E, L \cap E, \lambda)$ , where  $\lambda$  is restricted to  $L \cap E$ .

If  $f: [a, b] \rightarrow R$  be Riemann integrable, then  $f \in L_1[a, b]$  and  $\int f d\mu = \int_a^b f(x) dx$ . Mean Value Theorems.

**Total No. of Classes: 105. No tutorial class.**

Recommended Text: An Introduction to Measure and Integration—I.K.Rana

#### **(4) Special Theory of Relativity**

Review of Newtonian Mechanics—Inertial Frames. Speed of light and Galilean relativity. Michelson-Morley experiment. Lorentz- Fitzgerold contraction hypothesis. Relative character of space and time. Postulates of special theory of relativity. Lorentz transformation equations and its geometrical interpretation. Group properties of Lorentz transformations.

Relative kinematics—composition of parallel velocities. Length contraction. Time dilation. Transformation equations for components of velocity and acceleration of a particle and Lorentz contraction factor.

Geometrical representation of space-time—Four dimensional Minkowskian space-time of special relativity. Time-like, light-like and space-like intervals. Null cone. Proper time. World line of a particle. Four vectors and tensors in Minkowskian space-time, group property of Lorentz transformation and Lorentz Invariance.

Relativistic mechanics—variation of mass with velocity. Equivalence of mass and energy. Transformation equations for mass momentum and energy. Energy-momentum four vector. Relativistic force and Transformation equations for its component. Lagrangian Formulation of Classical Mechanics. Calculus of Variation, Principle of Least Action and Hamilton's Principle. Relativistic Lagrangian and Hamiltonian. Relativistic equations of motion of a particle. Energy momentum tensor of a continuous material distribution.

**Total No. of classes: 105. No tutorial class.**

- Book Recommended: (1) Classical Mechanics—H. Goldstein  
(2) Introduction to The Theory of Relativity—P.G.Bergmann  
(3) Introduction to Elementary Particles—D.Griffith

### **PAPER VII**

#### **Paper VII,Module 1** (Algebra-III)

**40 marks**

Inner Product Space(8).

Direct Product of Groups(4).

Conjugacy relation. Normaliser. The class equation of a finite group. Centre for group of prime order. Cauchy's Theorem and its applications; converse of Lagrange's Theorem for finite commutative group, Sylow's First Theorem, Sylow p-subgroup, statements of Sylow's Second and Third Theorems(20).

Polynomial Rings:  $R$  is an I D implies  $R[x]$  is an I D,  $R$  is a field implies  $R[x]$  is an ED,  $D$  is a UFD implies  $D[x]$  is a UFD. Sufficient condition of irreducibility of polynomials—Eisenstein's criterion(15).

Field Extensions(20).

**Total No. of classes for general teaching:52. No. of tutorial classes:25.**

- Books Recommended:** (1) Modern Algebra with Applications—W.J.Gilbert  
(2) Algebra—M. Artin  
(3) Topics in Abstract Algebra—M.K.Sen,S.Ghosh, P.Mukhopadhyay  
(4)Algebra- (4 Volume)—Luther and Pasi

**Paper VII, Module 2** (Programming in C Theory and Practical)

**20+40=60 marks**

Programming in C (Theory)

Computer fundamentals: Historical evolution ,computer generations ,functional descriptions ,operating systems, hardware and software **(1)**. Positional number systems: binary, octal, decimal, hexadecimal. Binary arithmetic**(2)**.

Storing of data in a computer : BIT, BYTE, Word. Coding of data – ASCII, EBCDIC, etc.**(2)**

Algorithm and Flowchart :Important features, Ideas about the complexities of algorithm. Application in simple problems.**(3)**

Programming languages: General concepts, Machine language, Assembly language ,High level languages. Compiler and Interpreter. Object and Source Program. Ideas about some major HLL.**(2)**

Character set in ANSI C. Key words : if, while, do, for, int, char, float etc.

Data type : character, integer ,floating point etc **(3)**.

Variables, operators,: = , = = , !! , < , > , etc.(arithmetic, assignment , relational , logical , increment , etc. )**(2)** .

Expressions : (a = = b) !! (b = = c ), Statements : e.g. if (a<b)small = a; else small = b**(2)**.

Standard input/output **(2)**.

Use of while , if .... Else, for, do .... While switch, continue etc.**(4)**

Arrays ,strings **(3)**

Function Definition **(4)**.

Running simple C Programs **(4)**.

Header Files **(2)**.

**Practical : Applications Of C in Numerical Analysis 40 marks**

Numerical Integration: Trapezoidal, Simpson's 1/3- rule, Weddle's rule and Romberg method of integration.

Numerical solution of non-linear equations: Tabulation,Bisection,Secant/ Regula Falsi, Fixed point iteration method, Newton-Raphson method and Graeffe's root squaring method.

Numerical solution of a system of linear equations: Gauss, Gauss-Jordan elimination method, Gauss-Jacobi, Gauss- Seidel iteration method. Matrix inversion by Gauss method.

Power methods for finding the extreme eigenvalues (3x3 or 4x4 order).

Numerical solution of ordinary differential equation—Euler's method, Picard method, Runge-Kutta method (fourth order).

Numerical solution of partial differential equation.

*(Above problems are to be done on computer using C language)*

**Total No. of general classes: 37. No of Practical classes 40.**

**Books Recommended** (1) Numerical Methods—E. Balagurusamy

(2) Let Us C—Y. Kanetkar

(3) C Programming—B.S.Gottfried

(4) Programming in C—E. Balagurusamy

**PAPER-VIII**

**Paper VIII, Module 1** (Differential Equation II AND Complex Analysis)

**50 marks**

## Differential Equation

**25 marks**

Total differential equations: condition of integrability(3), Partial Differential Equation(PDE) : Introduction, Formation of PDE. Solution of PDE by Lagrange's Method of solution and by Charpit's Method(12).

Laplace Transform and Inverse Laplace Transform. Statement of existence theorem. Elementary properties of Laplace Transform and its inverse. Laplace Transform of derivatives. Laplace Transform of integrals. Convolution Theorem (statement only). Application to the solution of differential equations of second order (simple problems only) (12).

Series solution at an ordinary and regular singular point: Power Series solution of ordinary differential equations, simple problems only(8).

## Complex Analysis

**25 marks**

Field structure of complex numbers, field of complex numbers can not be totally ordered(2), Geometric Interpretation of complex numbers(4), Topology of the complex plane(3), sequence and series of complex numbers(2).

Functions of a complex variable—Exponential, Logarithmic, Direct and Inverse Circular and Hyperbolic Functions(5). Injective and Surjective functions(1). Concepts of limit and continuity, sequential continuity is equivalent to continuity, continuity and connectedness, continuous function on a compact set is uniformly continuous(5).

Stereographic Projection(2).

Analytic Functions: Differentiability—definition, derivability implies continuity, differentiability of sum, difference, product, quotients and composition of differentiable functions, satisfaction of Cauchy-Riemann equations is necessary but not sufficient conditions for differentiability of a function at a point in its domain of definition, sufficient conditions for differentiability. Definition of Analytic and Entire function(6).

Harmonic function. Harmonic conjugate. The real and imaginary parts of an analytic function defined on an open subset  $O$  of the complex plane are harmonic on  $O$ . Statement of the existence of a harmonic conjugate  $f$  on a simply connected domain  $O$ —example of construction of the same when (1)  $f$  is real valued harmonic polynomial in  $C$ , (2)  $f$  is real valued harmonic function in  $O$  where  $O$  is either an open disk or open rectangle(5).

Power Series as an Analytic function: radius of convergence of a power series. Absolute and uniform convergence of a power series strictly within the circle of convergence. The Root and Ratio tests. A power series and its derived power series have same radius of convergence. A power series is an analytic function strictly within its circle of convergence (statement only) and conversely if  $f$  is analytic in a domain  $D$ , then  $f$  can be represented by a power series locally about each point  $z_0$  in  $D$ .

**Total No. of classes: 77. No tutorial class.**

- Book Recommended: (1) Foundations of Complex Analysis—S. Ponnusamy  
(2) Differential Equations—Shepley L. Ross  
(3) Elementary Treatise on Laplace Transform—B. Sen  
(4) Elements of Partial Differential Equations—I.N. Sneddon  
(5) Differential Equations with Applications—G.F. Simmons

## **Paper VIII, Module 2** (Optional II)

**50 marks**

Any **one** from the following topics (1), (2), (3) and (4):

## (1) Mathematical Statistics

### Theory

Random Sample. Concept of sampling and various types of sampling. Sample and population. Collection, tabulation and graphical representation. Grouping of data, sample characteristic and their computation. Sampling distribution of a statistic. Estimates of a population characteristic or parameter. Unbiased and consistent estimates. Sample characteristics as estimates of the corresponding population characteristics. Sampling distributions of the sample mean and variance. Exact sampling distributions for the normal populations.

Bivariate samples. Scatter diagram. Sample correlation co-efficient. Least square regression lines and parabolas. Estimation of parameters. Method of maximum likelihood. Applications to Binomial, Poisson and Normal population. Statistical Hypothesis. Simple and composite hypothesis. Best critical region of a test. Neyman- Pearson theorem (statement only) and its application to Normal population. Likelihood ratio testing and its application to Normal population. Simple applications of hypothesis testing.

### Practical

Sample characteristics—mean, variance, skewness, kurtosis, excess, mode, median, semi-interquartile range. Bivariate samples—correlation coefficient, regression lines, parabolic curve fitting, goodness of fit. Confidence intervals for mean and standard deviation of a normal population. Approximate confidence limits for the parameter of a binomial population.

Tests of hypothesis—tests on mean and standard deviation of a normal population, comparison of means and standard deviations of two normal populations. Approximate tests on the parameter of a binomial population, on comparison of two binomial populations. Poisson distribution.

**Total No. of classes: 77. No tutorial class.**

## (2) Functional Analysis

Normed Space—definition and examples, norm is a continuous function, metric induced by norm and its translation invariance. Banach space: definition and examples, a subspace  $Y$  of a Banach Space  $X$  is complete iff  $Y$  is closed in  $X$ . Completion of a normed space.

Finite dimensional normed spaces and subspaces—every finite dimensional subspace of a normed space  $X$  is closed in  $X$ , equivalence of norms on a finite dimensional normed space. In a finite dimensional normed space  $X$ , any subset  $M$  of  $X$  is compact iff  $M$  is closed and bounded. F.Riesz's lemma. If the closed unit ball in a normed space  $X$  is compact, then  $X$  is finite dimensional.

Linear Operator—definition and examples, properties. Inverse operator.

Bounded and continuous linear operators—definition and examples. If a normed space  $X$  is finite dimensional, then every linear operator on  $X$  is bounded. A linear operator  $T$  between two normed spaces is (1) continuous iff bounded, (2) continuous iff continuous at a single point

Inner Product Space and Hilbert Space—definition and examples. Schwarz and Triangle inequality. Continuity of inner product. Completion of an inner product space. Subspace  $Y$  of a Hilbert Space  $H$  is complete iff  $Y$  is closed in  $H$ ; IF  $y$  IS FINITE DIMENSIONAL, THEN  $y$  IS COMPLETE; IF  $h$  IS SEPARABLE, SO IS  $y$ . Orthogonal complements and Direct Sums. Existence of complement of a closed subspace of a Hilbert Space. If  $Y$  is a closed subspace of a Hilbert Space  $H$ , THEN  $Y^\perp = H$ . Orthogonal and orthonormal sets in an inner product space. An orthonormal set is linearly independent. Bessel inequality. Gram-Schmidt process. Series related to orthonormal sequences and sets.

**Total No. of classes: 77. No tutorial class.**

Recommended Text: Introductory Functional Analysis with Applications: E. Kreyszig

### **(3) Mathematical Logic**

Informal remarks on Formal Languages.

#### Sentential Logic

The Language of Sentential Logic—connectives symbols, sentence symbols, expression, well-formed formula. Convention about omitting parenthesis. Induction Principle. Truth assignments—its uniqueness (statement only), Tautology, Truth Tables, Compactness Theorem (statement only).

#### First Order Logic

First Order Languages—Logical symbols, Quantifiers, Predicate Symbols, constant symbols, function symbols. Examples of First Order Language.

Expressions—terms—atomic formula—well formed formulas. Free Variables.

Truth and Models. Structure—definition and examples. Definition and examples of logical implication of a wff from a set of wffs. Logically equivalent wffs. Valid wff.

Deductive calculus : definition of deduction and Theorem; Induction Principle. Substitution: term substitutable for variable in a wff, Tautology.

Deductions and Metatheorems: Generalizing Theorem, Deduction Theorem, Contraposition, Reduction ad absurdum—examples.

Soundness and Completeness Theorems (statements only) and their corollaries. Compactness Theorem.

#### Undecidability

Let  $\text{Th } \mathbf{N}$  be the theory of the structure  $(\mathbf{N}; 0, S, <, E)$  where  $\mathbf{N} = \{0, 1, 2, \dots\}$ ,  $S$  is the successor function,  $E$  is the exponential function.

Result : Let  $A \subseteq \text{Th } \mathbf{N}$  be a set of sentences true in  $\mathbf{N}$ , and assume that the set  $\alpha / \alpha \in A$  of Godel numbers of members of  $A$  is a set definable in  $\mathbf{N}$ . Then we can find a sentence  $\sigma$  such that  $\sigma$  is true in  $\mathbf{N}$  but  $\sigma$  is not deducible from  $A$ .

#### Arithmetization of Syntax—Godel Numbers

Incompleteness and Undecidability: Fixed Point Lemma (statement only), Tarski Undecidability Theorem (The set  $\# \text{Th } \mathbf{N}$  is not definable in  $\mathbf{N}$ ),  $\# \text{Th } \mathbf{N}$  is not recursive. Godel Incompleteness Theorem: If  $A \subseteq \text{Th } \mathbf{N}$  and  $\#A$  is recursive, then  $\text{Cn } A = \{\alpha / \alpha \text{ is true in } A\}$  is not a complete Theory (assuming relevant result).

**Total No. of classes: 77. No tutorial class.**

Recommended Text: (1) A Mathematical Introduction to Logic—H.B. Enderton

(2) Completeness, Compactness and Undecidability—An Introduction to Mathematical Logic—A.B. Manaster.

### **(4) Application of Mathematics in Investment Science**

1. Introduction ( Cash flows, Investments and Markets, Typical Investment Problems)
2. The Basic Theory of Interest (Principal and Interest, Present Value, Present and Future Values of Streams, Internal Rate of Return, Evaluation Criteria, Applications and

Extensions).

3. Fixed Income Securities (The Market for Future Cash, Value Formulas, Bond Details, Yield, Duration, Immunization, Convexity).
4. The Term Structure of Interest Rates (The Yield Curve, The Term Structure, The Forward Rates, The Term Structure Explanations, Expectations Dynamics, Running Present Value, Floating Rate Bonds, Duration, Immunization).
5. Applied Interest Rate Analysis (Capital Budgeting, Optimal Portfolios, Dynamic Cash Flow Processes, Optimal Management, Valuation of a Firm)
6. Mean- Variance Portfolio Theory (Asset Return, Random Variables, Random Returns, Portfolio Mean and Variance, The Feasible Set, The Markowitz Model, The Two Fund Theorem, Inclusion of a Risk-Free Asset, The One-Fund Theorem)
7. The Capital Asset Pricing Model (Market Equilibrium, The Capital Market Line, The Pricing Model, The Security Market Line, Investment Implications, Performance Evaluation)
8. Models and Data (Factor Models, The CAPM as a Factor Model, Data and Statistics, Estimation of other Parameters)

**Total No. of classes: 77. No tutorial class.**

Recommended Text: Investment Science—David Luenberger (Oxford University Press)

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SYLLABUS FOR THE THREE-YEAR GENERAL COURSE IN MATHEMATICS

**Course Structure**

**First Year**

Semester I

**Module I:** Classical Algebra & Differential Calculus (75 marks)

Semester II

**Module II:** Modern Algebra, Integral Calculus, Differential Equations (75 marks)

**Second Year**

**Module III:** Geometry, Vector Algebra, Numerical Analysis (75 marks)

**Module IV:** Linear Programming and Optional (75 marks)

**Third Year**

**(Only for General Course students. Two optionals to be chosen from a set of three topics)**

**Module V :** Optional 1( 50 marks)

**Module VI:** Optional 2 (50 marks)

**Availability of classes (minimum) per module:**

Year	Sem	No. of weeks available for teaching per semester	No. of General classes/week	Total no. of Hons. Classes/sem	No. of module/sem	No.of classes/module
I	I	15	6	90	1	90
	II	15	6	90	1	90
II	I	15	6	90	1	90
	II	15	6	90	1	90
III	I	15	6	90	1	90
	II	11	6	66	1	66

## Detailed Syllabus

(Figures in the margin indicate the number of lectures recommended to cover the topic)

### FIRST YEAR

#### **MODULE-I** (Classical Algebra and Differential Calculus)

**75 marks**

##### Classical Algebra (25 marks—20 classes)

1. Complex Numbers: De Moivre's Theorem and its applications(4). Exponential, Sine, Cosine and Logarithm of a complex number. Definition of  $a^z$  ( $a > 0$ ). Inverse circular and Hyperbolic functions (5)
2. Polynomials : Fundamental Theorem of Classical Algebra (statement only). Polynomials with real co-efficients : the  $n$  th degree polynomial equation has exactly  $n$  roots(1). Nature of roots of an equation (surd or complex roots occur in pairs)(1). Statement of Descarte's Rule of signs and its applications(2). Statement of : (1) If the polynomial  $f(x)$  has opposite signs for two real values of  $x$ , e.g.  $a$  and  $b$ , the equation  $f(x) = 0$  has an odd number of real roots between  $a$  and  $b$ ; if  $f(a)$  and  $f(b)$  are of same sign, either no real root or an even number of roots lies between  $a$  and  $b$ (1), (2) Rolle's Theorem and its direct applications(1). Relation between roots and co-efficients. Symmetric functions of roots(2), Transformation of equations(2). Cardan's Method of solution of a cubic(1).

##### Differential Calculus (50 marks—65 classes)

1. Rational Numbers. Geometrical Representations. Irrational number. Real Number represented as point on a line—linear combination. Acquaintance with basic properties of real number (no deduction or proof is included)(2).
2. Sequence: definition of bounds of a sequence and monotone sequence. Limit of a sequence. Statements of limit theorems. Concept of convergence and divergence of monotonic sequences—applications of the theorems, in particular, definition of  $e$ . Statement of Cauchy's General Principle of convergence and its applications(5).
3. Infinite Series of constant terms: convergence and divergence (definitions). Cauchy's Principle as applied to infinite series (application only). Series of positive terms: Statements of Comparison Test, D'Alembert's Ratio Test, Cauchy's  $n$  th Root Test and Raabe's Test—applications. Alternating series: statement of Leibnitz Test and its applications.(5)
4. Real valued function defined on an interval: limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval. Acquaintance (no proof) with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotone function and its continuity.(8)
5. Derivative—its geometrical and physical interpretation. Sign of derivative—monotone increasing and decreasing functions. Relations between continuity and derivability. Differential – application in finding approximation.(4)
6. Successive derivatives—Leibnitz Theorem and its applications(5)
7. Statement of Rolle's Theorem and its geometrical interpretation. Mean Value Theorems of Lagrange and Cauchy. Statement of Taylor's and Maclaurin's Theorem

- with Lagrange's AND Cauchy's form of remainders. Taylor's and Maclaurin's infinite series for functions like  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $(1+x)^n$ ,  $\log(1+x)$  (with restrictions wherever necessary).(8)
8. Indeterminate Forms: L'Hospital's Rule: statement and problems only.(2)
  9. Application of the principle of maximum and minimum for a function of a single variable in geometrical, physical and other problems.(3)
  10. Functions of two and three variables: their geometrical representations. Limit and continuity (definitions only) for functions of two variables. Partial derivative: knowledge and use of chain rule. Exact differentials (emphasis on problem solving only). Functions of two variables—successive partial derivatives: statement of Schwarz's theorem on commutativity of mixed derivatives. Euler's Theorem on homogeneous function of two and three variables. Maxima and minima of functions of not more than three variables—Lagrange's method of undetermined multiplier—problems only. Implicit function in case of functions of two variables (existence assumed) and derivative.(8)
  11. Applications of Differential Calculus: Tangents and Normals, Pedal equation and Pedal of a curve. Rectilinear Asymptotes (Cartesian only). Curvature of plane curves. Envelope of family of straight lines and of curves (problems only). Definitions and examples of singular points (viz. Node, Cusp and Isolated Points).(15).

**MODULE –II** (Modern Algebra, Integral Calculus, Differential Equation)

**75 marks**

Modern Algebra (35 marks(15: abstract algebra, 20:linear algebra)—45 classes)

1. Basic concepts: Sets, subsets, equality of sets. Operations on sets: union, intersection and complement. Verification of the laws of algebra of sets and De Morgan's Laws. Cartesian product of two sets.(3)
2. Mappings, injective and surjective mappings(2). XComposition of mappings—concept only(1). Identity and inverse mappings. Binary operations on a set. Identity element. Inverse elements(2).
3. Introduction to Group Theory—definition and examples taken from various branches (examples from number system, roots of unity,  $2 \times 2$  real matrices, non-singular real matrices of a fixed order) (2). Elementary properties using definition of groups(1). Definition and examples of subgroup—statement of necessary and sufficient condition—its applications(2) .
4. Definition and examples of (1) Ring, (2) Field, (3) subring, (4) subfield.(3)
5. Determinants of upto third order: properties , cofactors and minors(2). Product of two determinants. Adjoint, Symmetric and skew-symmetric determinants(2). Solutions of linear equations with not more than three variables by Cramer's Rule.
6. Matrices of real numbers: equality of matrices. Addition of matrices. Multiplication of matrices by a scalar. Multiplication of matrices—associative property(2). Transpose of a matrix—its properties(1). Inverse of a non-singular square matrix(2). Symmetric and non-symmetric matrices(1). Scalar matrix. Orthogonal Matrix(1).

Elementary operations on matrices(3). Rank of a matrix: determination of rank either by considering minors or by sweep-out process(1) . Consistency and solution of a system of linear equations with not more than three variables by matrix method(2).

7. Concept of Vector Space over a field—examples(2), concepts of linear combinations, linear dependence and independence of a finite set of vectors(2), subspace(1), concepts of generators and basis of a finite dimensional vector space. Problems on formation of a basis of a vector space (no proof required)(4)
8. Real Quadratic form involving not more than three variables—problems only.(2)
9. Characteristic equation of a square matrix of order not more than three (1)—determination of eigenvalues and eigenvectors—problems only(2). Statement and illustration of Cayley- Hamilton Theorem(2).

Integral Calculus and Differential Equations (20+20 = 40 marks ; 50 classes)

1. Integration of the form:  $\int \frac{dx}{a + b \cos x}$ ,  $\int \frac{l \sin x + m \cos x}{n \sin x + p \cos x} dx$  and integration of rational functions (2).
2. Evaluation of definite integrals(2).
3. Integration as the limit of the sums (with equally spaced as well as unequal intervals)(2).
4. Reduction Formulae of  $\int \sin^m x \cos^n x dx$ ,  $\int \tan^n x dx$  and associated problems (m and n are non-negative integers)(4)
5. Definition of improper integrals: statement of (1)  $\mu$ -test, (2) comparison test (limit form excluded)—simple forms only. Use of Beta and Gamma functions (convergence and important relations being assumed) (5).
6. Working knowledge of Double Integrals(4).
7. Applications: Rectification, Quadrature, Volumes and Surface Areas of solids formed by revolution of plane curve and areas—problems only (4).
8. Order, Degree and solution of an Ordinary Differential Equation (O.D.E.) in presence of arbitrary constants. Formation of O.D.E.(5).
9. First order equations : (1) variables separable, (2) homogeneous equations and equations reducible to homogeneous forms, (3) exact equations and those reducible to such equation(7), (4) Euler's and Bernoulli's equation (linear)(3), (5) Clairaut's equation: general and singular solutions(3).
10. Second order linear equations: second order linear differential equations with constant co-efficients. Euler's homogeneous equations(6).

11. Simple applications: Orthogonal Trajectories(2)

**SECOND YEAR**

**MODULE III** (Geometry, Vector & Numerical Analysis)

**75 marks**

Geometry and Vector(50 marks, 55 classes)

1. Transformation of rectangular axes: translation, rotation and their combinations. Invariants(4).
2. General Equation of second degree in x and y: Reduction to canonical form. Classification of conics(4)
3. Pair of straight lines: condition that the general equation of second degree in x and y may represent two straight lines (1). Point of intersection of two intersecting straight lines(1). Angle between two lines given by  $ax^2+2hxy+by^2 = 0$ . Equations of bisectors. Equation of two lines joining the origin to the points in which a line meets a conic(2).
4. Equations of pair of tangents from an external point , chord of contact, poles and polars in case of general conic ; particular case for parabola, ellipse, circle , hyperbola(6)
5. Polar equations of straight lines and circles(3). Polar equation of a conic referred to a focus as pole(2). Equation of chord joining two points. Equations of tangents and normals(3).
6. Rectangular Cartesian co-ordinates: distance between two points. Division of a line segment in a given ratio(2). Direction cosines and direction ratios of a straight line(1). Projection of a line segment on another line(1). Angle between two straight lines(1).
7. Equations of a plane: general form, intercept and normal form(2). Angle between two planes(1). Signed distance of a point from a plane(1). Bisectors of angles between two intersecting planes(1). Problems(2).
8. Equations of straight lines: general and symmetric forms(2). Distance of a point from a line(1). Coplanarity of two straight lines(1). Shortest distance between two skew lines(2).
9. Sphere and its tangent plane(4).
10. Right circular cone(2)
11. Addition of vectors, multiplication of vector by a scalar(2). Collinear and coplanar vectors(3). Scalar and vector products of two and three vectors(2). Simple applications to problems of Geometry(2). Vector equation of plane and straight

line(3). Volume of tetrahedron(1). Application to problems of Mechanics ( work done and moment)(1)

Numerical Analysis(25 marks,30 classes)

1. Approximate numbers, significant figures, rounding off numbers. Error—absolute, relative and percentage (2).
2. Operators--  $\Delta, \nabla, E$ ( definitions and some relations among them) (3)
3. Interpolation: the problem of interpolation. Equi-spaced arguments—difference tables, deduction of Newton's Forward Interpolation Formula, Remainder Terms (expression only). Newton's Backward Interpolation Formula, Remainder Terms (statement only). Unequally spaced arguments—Lagrange's Interpolation Formula (statement only). Numerical problems on interpolation with both equi- and unequally-spaced arguments(7).
4. Numerical Integration: Trapezoidal and Simpson's 1/3 rd formula (statement only). Problems on numerical integration(3).
5. Solution of equation: To find real root of an algebraic or transcendental equation. Location of root (tabular method), bisection method, Newton-Raphson method with geometrical significance, numerical problems(7).

**Note:** Emphasis should be given on problems.

**MODULE IV** (Group A **COMPULSORY** and **any one** of Group B and Group C) **75 marks**

**GROUP-A** (Linear Programming—35 marks, 40 classes)

Motivation of Linear Programming Problem. Statement and formulation of L.P.P.(3). Slack and surplus variables. L.P.P. in matrix form(2). Convex set, hyperplane, extreme points, convex polyhedron(4), basic solutions and basic feasible solutions (b.f.s.). Degenerate and non-degenerate b.f.s.(3).

The set of all feasible solutions of an L.P.P. is a convex set(1). The objective function of an L.P.P. assumes its optimal value at an extreme point of the convex set of feasible solutions. A b.f.s. to an L.P.P. corresponds to an extreme point of the convex set of all feasible solutions(5).

Fundamental Theorem of L.P.P.(statement only). Reduction of a feasible solution to a b.f.s.(3). Standard form of an L.P.P. Solution by graphical method(for two variables)(3), by Simplex method and method of penalty. Concept of duality(1). Duality theory. The dual of the dual is the primal(1). Relation between the objective values of dual and the primal problems(1). Dual problems with at most one unrestricted variable, one constraint of equality(4).

Transportation and Assignment problem and their optimal solution(7).

**GROUP-B** (Statistics and Probability, 40 marks, 50 classes)

1. Elements of Probability Theory: Random Experiment, outcome, event, mutually exclusive events, equally likely and exhaustive. Classical definition of probability. Theorems of Total Probability, conditional probability and statistical independence, Bayes' Theorem, problems. Shortcomings of the classical definition. Axiomatic approach—problems, random variables and its expectation. Joint distribution of two random variables. Theoretical Probability Distribution—discrete and continuous (p.m.f. and p.d.f.). Binomial, Poisson and Normal distribution and their properties(18).
2. Elements of Statistical Methods. Variables, attributes. Primary data, secondary data. Population and samples. Census and sample survey. Tabulation—chart and diagram, graph, bar diagram, Pi diagram etc. Frequency Distribution—ungrouped and grouped cumulative frequency distribution. Histogram, frequency curve, measures of central tendencies—averages; A.M., G.M., Mean, Median and Mode (their advantages and disadvantages). Measures of Dispersion: range, quartile, deviation, mean deviation, variance/ s.d., moments, skewness and kurtosis(8).  
Sampling Theory: meaning and objects of sampling. Some ideas of the methods of selecting samples. Statistic and parameter, sampling distribution—standard error of a statistic (e.g. sample mean, sample proportion). Four fundamental distributions derived from the Normal: (1) Standard Normal Distribution, (2) Chi-square Distribution, (3) Student's Distribution, (4) Snedecor's F-distribution. Estimation and Test of Significance. Statistical Inference. Theory of Estimation—point estimation and interval estimation. Confidence Interval/ Confidence Limit. Statistical Hypothesis—Null Hypothesis and Alternative Hypothesis. Level of Significance. Critical Region. Type I and Type II error. Problems (10)  
Bivariate Frequency Distribution. Scatter Diagram. Correlation Coefficient—definition and properties. Regression lines.(5)  
Time Series: definition. Why to analyze Time Series data?  
Components. Measurement of Trend—(1) moving average method, (2) curve fittings (linear and quadratic curve). Ideas of other curves, e.g. exponential curve etc. Ideas about the measurement of other components(6).  
Index Number: meaning of index number. Construction of Price Index Number. Consumer Price Index Number. Calculation of purchasing power of Rupee(3)

**GROUP-C** (Advanced Calculus, marks 40, 50 classes)

1. Concepts of pointwise and uniform convergence of sequence of functions and series of functions with special reference to Power Series. Statement of Weierstrass's M-test for uniform convergence of sequence of functions and of series of functions. Simple applications. Statement of important properties like boundedness, continuity, differentiability and integrability of the limit function of uniformly convergent sequence of functions and of the sum function of uniformly convergent series of functions. Determination of radius of convergence of Power Series. Statement of properties of continuity of sum function of power series. Term by term integration and term by term differentiation of power series. Statement of Abel's Theorem on

- power series. Convergence of power series. Expansion of elementary functions such as  $e^x$ ,  $\sin x$ ,  $\log(1+x)$ ,  $(1+x)^n$ . Simple problems.
2. Second Order Linear Differential Equation: (1) method of variation of parameters, (2) method of undetermined coefficients, (3) simple eigenvalue problem.
  3. Partial Differential Equation(PDE) : Introduction to PDE, solution of PDE, Lagrange's method of solution.

### **THIRD YEAR**

**Two Modules are to be taken from the following set of three Modules.**

**(One module in each semester)**

**MODULE V** (Elements of Computer Science and Programming)

**50 marks**

1. Boolean Algebra—Basic Postulates and definition. Two-element Boolean Algebra. Boolean function. Truth table. Standard forms of Boolean function—DNF and CNF. Miniterms and Maxterms.Principle of Duality. Some laws and theorems of Boolean Algebra. Simplification of Boolean expressions—algebraic method and Karnaugh Map method. Application of Boolean algebra—switching circuits, circuits having some specified properties, Logical Gates—AND,NOT,OR,NAND,NOR etc.
2. Computer Science and Programming: Historical development,computer generation, computer anatomy—different components of a computer system. Operating System, Hardware and Software.  
Positional Number System. Binary to Decimal and Decimal to Binary. Other Systems.Binary arithmetic. Octal, Hexadecimal etc. Storing of data in a computer—BIT,BYTE,WORD etc. Coding of data—ASCII etc.  
Programming Language: machine language, assembly language and High Level language. Compiler and Interpreter. Object Programme and Source Programme. Ideas about some HLL—e.g. BASIC,FORTRAN,C,C++,COBOL,PASCAL etc.  
Algorithms and Flow Charts—their utilities and important features, Ideas about the complexities of an algorithm. Application in simple problems. Fortran 77/90: Introduction, Data Type—keywords, constants and Variables—Integer, Real, Complex, Logical, Character, Subscripted Variables, Fortran Expressions.  
I/O statements—formatted and unformatted. Programme execution control—logical if, if-then-else etc. Arrays, dimension statement. Repetitive Computation-Do,Nested Do etc.Sub Programs—(1) Function subprogram, (2) Subroutine sub program.  
Elements of BASIC Programming Language: Reading,Printing,Branch & Loop, Array, Functions.  
Applications to simple problems. An exposure to M.S. Office, e-mail, Internet (through demonstration only)

**MODULE VI** (A Course of Calculus)

**50 marks**

1. Concepts of pointwise and uniform convergence of sequence of functions and series of functions with special reference to Power Series. Statement of Weierstrass's M-test for uniform convergence of sequence of functions and of series of functions. Simple applications. Statement of important properties like boundedness, continuity, differentiability and integrability of the limit function of uniformly convergent sequence of functions and of the sum function of uniformly convergent series of functions. Determination of radius of convergence of Power Series. Statement of

properties of continuity of sum function of power series. Term by term integration and term by term differentiation of power series. Statement of Abel's Theorem on power series. Convergence of power series. Expansion of elementary functions such as  $e^x$ ,  $\sin x$ ,  $\log(1+x)$ ,  $(1+x)^n$ . Simple problems.

2. Fourier Series on  $(-\pi, \pi)$ : Periodic function, determination of Fourier Coefficients. Statement of Dirichlet's conditions of convergence and statement of the theorem on convergence of Fourier Sine and Cosine series.
3. Third and Fourth order ordinary differential equation with constant coefficients. Euler's homogeneous equations.
4. Second order differential equation: (1) method of variation of parameters, (2) method of undetermined coefficients, (3) simple eigenvalue problem.
5. Simultaneous linear differential equation with constant co-efficients.
6. Partial Differential Equation(PDE) : Introduction to PDE, solution of PDE, Lagrange's method of solution.
7. Laplace Transform and its application to ordinary differential equation. Laplace transform and inverse Laplace transform. Statement of existence theorem. Elementary properties of Laplace transform and of its inverse. Application to the solution of ordinary differential equation with constant co-efficients.

## **MODULE VII** (Discrete Mathematics)

**50 marks**

1. Integers: Principle of Mathematical Induction. Division Algorithm. Representation of integers in an arbitrary base. Prime integers. Some properties of prime integers. Fundamental Theorem of Arithmetic. Euclid's Theorem. Linear Diophantine Equations. (Statement of Principle of Mathematical Induction, strong form of mathematical induction. Applications in different problems. Proof of division algorithm. Representation of an integer uniquely in an arbitrary base, change of an integer from one base to another base. Computer operations with integers. Divisor of an integer, gcd of two positive integers, prime integer. Proof of Fundamental Theorem. Proof of Euclid's Theorem. To show how to find all prime numbers less than or equal to a given positive integer. Problems related to prime numbers, linear Diophantine Equation—when such an equation has solution, some applications).
2. Congruences: congruence relation on Integers. Basic properties of this relation. Linear congruences, Chinese Remainder Theorem. System of linear congruences. (Definition of congruence—to show it is an equivalence relation, to prove the following:  $a \equiv b \pmod{m}$  implies (1)  $a+c \equiv b+c \pmod{m}$ , (2)  $ac \equiv bc \pmod{m}$ , (3)  $a^n \equiv b^n \pmod{m}$ , (4) for any polynomial  $f(x)$  with integral co-efficients,  $f(a) \equiv f(b) \pmod{m}$  etc. Linear congruence, to show how to solve these congruences, Chinese Remainder Theorem—statement and proof and some applications. System of linear congruences, when solution exist—some applications.)
3. Application of congruences: divisibility test, computer file, storage and hashing functions. Round-robin tournaments. Check-digit in an ISBN, in Universal Product Code, in major Credit Cards. Error detecting capability. ( Using congruence, develop divisibility tests for integers based on their expansions with respect to different bases. If  $d$  divides  $(b-1)$ , then  $n = a_k a_{k-1} \dots a_1 b$  is divisible by  $d$  iff the sum of the digits is divisible by  $d$  etc. Show that congruence can be used to schedule Round-robin Tournaments. A University wishes to store a file for each of its students in its computer. Systematic methods of arranging files have been developed based on

hashing function  $h(k) \equiv k \pmod{m}$ . Discuss different properties of this congruence and also problems based on this congruence. Check digits for different indication numbers—ISBN, Universal Product Code etc. Theorem regarding error detecting capability).

4. Congruence classes: congruence classes, addition and multiplication of congruence classes. Fermat's Little Theorem. Euler's Theorem, Wilson's Theorem. Some simple applications. (Definition of congruence classes, properties of congruence classes, addition and multiplication of congruence classes, existence of inverse. Fermat's Little Theorem. Euler's Theorem. Wilson's Theorem—statement, proof and some applications).
5. Recurrence Relation and Generating Functions: Recurrence Relation. The method of iteration. Linear Difference Equation with constant co-efficients. Counting with generating functions.
6. Boolean Algebra: Boolean Algebra, Boolean Functions, Logic Gates, minimization of circuits.

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