St. Xavier's College, Kolkata Department of Mathematics

PROPOSED DETAILED SYLLABUS FOR MATHEMATICS HONOURS COURSE, 2014

Total 1800 marks—18 papers, each carrying 100 marks and Exam duration 3hours (except Project MT36523) For each 100 marks paper, six classes available per week. For each semester, Twelve weeks available for class hours

Semester- I	Paper- I (MT 31011)	Algebra-I Section –I (50)
		Section –II (50)

Paper-II (MT31021)	Analysis-I Section –I (50)
	Section –II O.D.E (50)

- Semester-II Paper- III (MT32031) Algebra-II Section –I(50) Analysis-II Section- II (50)
 - Paper- IV (MT32041) Vector Algebra Section-I(50) 2D and 3D Geometry Section-II(50)
- Semester-III Paper- V (MT33051) Section-I : Algebra -III(50) Section-II : Analysis-III(50)

Paper-VI (MT 33061) Elements of Mechanics I (100)

- Semester-IV Paper-VII (MT34071) Section-I: Introduction to Computer Programming(20) Section –II: Numerical Analysis Theory (50) Section-III: Total Differential Equation and P.D.E.(30)
 - Paper-VIII **(MT34081)** Section-I : Algebra-IV (80) Section-II: Number Theory (20)

Paper-IX (MT34091) Metric Spaces (100)

	Paper-X (MT34101)	Section-I: Inner Product space (20)
		Section-II: L.P. & Game Theory (80)
Semester-V	Paper-XI (MT35511)	Practical Paper in Numerical Analysis through C Programming(100)
	Paper-XII (MT35111)	Section-I: Analysis III(50)
		Section-II: Analysis IV (50)
	Paper-XIII (MT35121)	Section-I: Probability Theory I(50)
		Section-II: Functions of Several Variable(50)
	Paper-XIV (MT35131)	Optional I(Topology/ Theory of Special Relativity)(100)
Semester-VI	Paper-XV (MT36141)	Section-I: Vector Analysis(50)
		Section-II: Elements of Mechanics II (50)
	Paper-XVI (MT36151)	Section-I: Algebra-V(50)
		Section-II: Complex Analysis(50)
	Paper-XVII (MT36161)	Optional II (Elementary Differential Geometry/Probability Theory II and
		Statistics/Functional Analysis)(100)
	Paper-XVIII (MT36523)	Dissertation/Project and Grand Viva(100)

DETAILED SYLLABUS

PAPER-I (Algebra-I) (MT 31011)

SECTION-I (50marks)

36 classes

Course Objective: Learning and application of : 1. simplified expression of z^n , z complex, n rational, 2. matrix and determinant (over R or C) of general order, their properties and evaluation of determinant 3. processes to find or locate roots of a polynomial equation.

- **Complex numbers**: Recapitulation of complex number system, modulus-amplitude form, Argand diagram, De Moivre's Theorem and its applications. [5]
- Theory of Equations: Polynomials with real or complex coefficients, Fundamental Theorem of Classical Algebra (statement only), synthetic division, multiple roots (4).Relations between the roots and coefficients. Symmetric functions of roots(2). Descarte's Rule of signs, Sturm's Theorem—statements and applications (3). Transformations of equations (2) .Cardan's method of solution of a cubic equation and Ferrari's method of solution of a biquadratic equation(3).[15]

• Matrix &Determinant: Matrices over a field [R or C]; Operations of matrices and their properties (2), Special types of matrices (square, identity, diagonal, triangular, singular and non-singular), transpose of a matrix—symmetric, skew-symmetric, Hermetian, skew Hermetian, orthogonal and unitary matrices, Trace of a matrix (5).Definition and statement of properties of determinant (4).Cofactor, Minor, Jacobi's First and Second Theorems—statement and application (2), Laplace's expansion. Classical adjoint and inverse of a matrix (2). [16]

SECTION-II (50marks)

Course Objective: Learning and application of : 1. concept of equivalence relation and partition on a set and their interrelation, 2. concept of internal binary operation and algebraic systems: in particular, with group, 3. Lagrange's Theorem relating order of a subgroup of a finite group to that of the group.

Sets and their Cartesian product (2), Relations, Equivalence Relations and Partitions (4). Mappings (4). Binary operation, Algebraic system with special reference to field as an example of an algebraic system. Definition, examples, properties of groups (8), groups of n th roots of unity, permutation groups, group of residue modulo classes (5), properties relating to order of an element of a group, order of a group (3), Subgroups(3), Cyclic groups(3), Cosets (2), Lagrange's Theorem for finite groups(2) [36]

Books Recommended:

- (1) The Theory of Equations —Burnside, Panton
- (2) Higher Algebra—Barnard and Child.
- (3) Higher Algebra (Classical, Abstract& Linear) S. K. Mapa
- (4) Modern Algebra—Surjeet Singh and Qazi Zameruddin
- (5) First Course in Abstract Algebra— J. B. Fraleigh
- (6) Abstract Algebra—D.S. Dummit and R. M. Foote
- (7) Algebra—M. Artin
- (8) Topics in Algebra—I. N. Herstein
- (9) Topics in Abstract Algebra-M. K. Sen, S. Ghosh, P. Mukhopadhyay
- (10) Elementary Linear Algebra—Howard Anton, Chris Rorres
- (11) Linear Algebra- K. B. Datta

PAPER-II (Analysis-I) (MT 31021)

SECTION-I (50marks)

(36 classes)

Course Objective: Learning and application of 1. axiomatic definition of real number system, in particular, with completeness, 2. bounded monotone sequence of real numbers and their convergence 3. Cauchy's Limit Theorems and 3. topology of real number system.

Real numbers: Field axioms for real numbers and other salient properties taken as axioms. Arithmetic Continuum. Concept of ordered Field. Concept of point set in one dimension. Bounded set. Least upper bound axiom or completeness axiom. Archimedean property and density property. Cantor-Dedekind Axiom, Symbols $+\infty$ and $-\infty$. Symbols of intervals. (13)

36 classes

Real Sequences: Bounds, Limits, Convergence & non-convergence of sequences. Operations on limits, Sandwich rule, Monotone sequences and their convergence (Statement and proof either for increasing sequence or for decreasing sequence, only statement for the other type). Nested Interval Theorem—statement & proof. Limits of some important sequences with special reference to that for limit "e". Cauchy's first and second limit theorems. (13)

Point set in one dimension:

Neighborhood of a point, Interior point. Accumulation point and isolated point of a linear point set. Bolzano-Weierstrass Theorem on accumulation point, Derived set. Open set and closed set. Union, Intersection & Complement of open & closed sets in R. No non- empty proper subset of R is both open and closed in R. Closure of a set to be defined as the union of the set & its derived set. Interior of a set S as the set of all its interior points. Deduction of basic properties of closure of a set and interior of a set.(10)

Books Recommended:

- (1) Introduction to Real Analysis—Bartle, Sherbert
- (2) Calculus (Vol. I)-T.M.Apostol
- (3) Undergraduate Analysis—S. Lang
- (4) Mathematical Analysis-S. C. Malik and Arora
- (5) Advanced Calculus(An Introduction to Classical Analysis) Louis Brand (Dover)
- (6) A First Course in Real Analysis—S. K. Berberian
- (7) Advanced Calculus—D. Widder
- (8) Mathematical Analysis—Elias Zakon

SECTION-II

(Ordinary Differential Equations) (50marks)

36 classes

Course Objective: Learning and application of : 1. formation of ODE from physical, geometrical and economical problems, 2. solution of first order first degree ODE with special reference to exact and reducible to exact ode, 3.solution of first order and higher degree ODE with special reference to singular solution, 4. different methods of solving linear higher order ODE, 5. methods of solving simultaneous linear first order ODE by matrix method.

Ordinary differential equations: significance of ordinary differential equations: Formation of differential equation by eliminating arbitrary constants. Solution of ordinary differential equation—definition and examples (3). Equations of first order and first degree : statement of existence theorem(1). Exact differential equation of first order— statement of necessary and sufficient conditions for exactness. Solving exact ode by (i) Grouping method & (ii) Method of partial integration(3). Separable & Homogeneous first order equations (1). Integrating factor: Different rules of finding integrating factors of a non-exact first order ode and related problems (3). First order Linear ode . Equations reducible to the first order linear ode (2). First order higher degree equations solvable for x, y, p (3) Clairaut's equation: singular solution and its envelope interpretation. Transformation of a first order differential equation to Clairaut's form (3).

Explicit methods of solving higher order linear differential equations: Basic Theory of linear differential equations & Symbolic Operator D, Problems based on shortcut methods for finding particular integrals (4). D'Alembert's order reduction method (2). Method of undetermined coefficients (3) Method of Variation of parameters(2), Simple eigen-value problems(3). Simultaneous linear differential equations and their solutions by matrix method (3).

Books Recommended:

- (1) Differential Equations—S. L.Ross
- (2) Differential Equations—G.F. Simmons & S. Krantz
- (3) Introduction to Differential Equations—Boyce & Diprima.
- (4) Mathematical Methods—Potter & Goldberg

PAPER-III(MT 32031)

SECTION-I (Algebra-II) (50 Marks) 36 classes Course Objective: Learning and application of : 1. concept of Vector Space, linear dependence and independence of a finite set of vectors, basis and dimension of vector space, 2. matrices as linear transformation, 3. finding rank and inverse of matrix using elementary operations, 4. checking consistency and process of solution of system of linear equations.

- Definition and examples of Vector Spaces (4). Subspaces: necessary and sufficient condition(3). Linear Span. Linear Dependence, independence and their properties(6). Basis and dimension, computation of basis, Infinite dimensional vector spaces : only examples(5).[18]
- Matrix: Linear Map –definition and examples, kernel and image of a linear map—related results(4). Nullity and rank, related results(3). Introduction of matrices as Linear transformation (1). Row space and column space of a matrix. Row rank, column rank, determinant rank and their equality. Rank of product of two matrices (no proof) (4). Elementary operation on matrices, Rank and inverse of a matrix. Row reduction (3). System of linear equations –consistency, matrix method of solution (3). [18]

Books Recommended:

- (1) Linear Algebra--a Geometric Approach -- S. Kumaresan
- (2) Linear Algebra- Freidberg, Insel, Spence
- (3) Linear Algebra-Rao, Bhimasankaram
- (4) Linear Algebra, Concepts and Applications— P.K.Nayak
- (5) Linear Algebra, an Introductory Approach— C. W. Curtis

SECTION-II

Analysis-II (50 marks)

36 classes

Course Objective: Learning and application of 1. concept of subsequential convergence, limit superior, limit inferior 2.different forms of completeness of real number system and their equivalence 3.Cauchy's general principle of convergence, 3. absolute and conditional convergence of series of real numbers and related tests, 4. limit of real valued function of real variable.

Sub-sequences in R: All subsequences of a convergent sequence converge to the same limit as that of original sequence. Every bdd sequence has monotone subsequence. Every bounded sequence has a convergent subsequence, Sub-sequential limits and related results. Lim sup & Lim inf and related result. Equivalence of BW and NIP. Statement of Cauchy's general principle of convergence.(12)

Infinite series of real numbers: Convergence, divergence, Cauchy's criterion of convergence. Abel– Pringsheim's Test*. Series of non-negative real numbers: Tests of convergence – Cauchy's condensation test*. Upper limit and lower limit criteria for (i) Root test*(ii) Kummer's test*. Statements of Ratio Test, Raabe's test, Logarithmic test and Gauss's test. Simple Problems.

Series of arbitrary terms: Absolutely convergent and conditionally convergent series. Alternating series: Leibnitz test*. Root test and Ratio test, Non-absolute convergence – Abel's and Dirichlet's test (Statements and applications) Rearrangement of series through examples. Riemann's Rearrangement theorem (Statement) and simple examples. Rearrangement of absolutely convergent series(Statement).(**Proof of only * marked results**) (16)

Real valued functions defined on set: Bounded functions, step functions, Monotone functions, Limits of functions : Algebra of limits and Sandwich rule (statement only).Limit of Monotone functions. Statement of Cauchy Criteria for the existence of finite limit. Standard Limits.(8)

Books Recommended:

- (1) Introduction to Real Analysis—Bartle, Sherbert
- (2) Calculus (Vol. I)—T. M. Apostol
- (3) Mathematical Analysis—S. C. Malik and Arora
- (4) Advanced Calculus (An Introduction to Classical Analysis)– Louis Brand (Dover)
- (5) A First Course in Real Analysis—S. K. Berberian

(6) Basic Real Analysis— Houshang H. Sohrab

PAPER-IV(MT32041)

SECTION-I Vector algebra (50 marks)

36 classes

Course Objective: Learning and application of : 1. concept of free vectors and their operations, 2. concept of linear dependence, independence, collinearity and coplanarity of vectors, 3.scalar and vector product and their

use in proof of trigonometric and geometrical results, 4. scalar and vectorial equation of straight lines and planes.

Localized & Free vectors. Characterisation of free vector by Parallelogram rule. Coplanarity and collinearity of vectors, Null vectors. Group of free vectors under vector addition. (2) Position vectors, Section ratio, Proof of concurrence of medians of a triangle (1), Linear dependence and independence of vectors in 2D & 3D, Theorems on coplanarity and collinearity of points(3). Scalar product of two vectors and its properties, Geometrical interpretation of scalar product as projection, Cauchy -Schwarz inequality, Vector product of two vectors and its properties, Polar and Axial vectors and their vector product . Application of scalar and vector product in proof of Trigonometric and Geometrical results. Vectorial area of a triangle and two-sidedness of a surface (5). Box product of three vectors and its properties,-right and left handed triad, Coplanarity of vectors and box product, Cramer's rule for solving system of linear equations from the viewpoint of box product(3), Moment of a localized vector about a point and a line (2), Vector triple product (proof included) and its properties(2), Vector equations (3) Straight lines and Planes(in details) (15)

SECTION-II

2D and 3D Geometry (50 marks)

36 classes

Course Objective: Learning and application of : 1. reduction of General equation of second degree in two variables to its canonical form by Method of Invariants, 2. pair of straight lines, 3. polar equation of conics, 4. Conicoids, in particular, Sphere, Cone, Cylinder.

Two dimensional Geometry: General equation of second degree in two variables and reduction to canonical form by Method of Invariants.(5),Pair of Straight lines(6),Tangent-normal, Conjugate diameters & Rectilinear Asymptotes (5) .Polar equation of conics (5) .[21]

Three dimensional Geometry: Sphere, Cone, Cylinder (10), Introduction to conicoids and their generating lines, Tangent -Normal (5) .[15]

Books Recommended (both sections combined):

- (1)Vector Analysis— J. C. Tallack
- (2)Elementary Vector Analysis—C. E. Weatherburn
- (3) Vector and Tensor Analysis—U. Chatterjee & N. Chatterjee
- (4) Calculus and Analytic Geometry-G. B. Thomas and R. L. Finney
- (5) Elements of Coordinate Geometry -S. L. Loney
- (6) Co-ordinate Geometry of Three Dimensions— J. T. Bell
- (7) Advanced Analytical Geometry of Two and Three Dimensions-U. Chatterjee & N. Chatterjee

PAPER-V(MT33051)

SECTION-I (Algebra -III) (50marks)

Course Objective: Learning and application of : 1. concept of quotient group, homomorphism and isomorphism of groups, isomorphic class of a group 2. First, Second and Third Isomorphism Theorems, 3. Cayley-Hamilton Theorem, diagonalisation of matrices, 4. reduction of Quadratic Forms to canonical form.

SUBSECTION-I

Normal subgroups, Quotient group (6). Homomorphism and Isomorphism of group--definition and examples(2) Homomorphism theorems relating to identity, inverse, image and inverse image of a subgroup, order of an image of an element(3). Kernel of a homomorphism—related results(2). Monomorphism, epimorphism, isomorphism—related results; isomorphic class of a group, examples (3). Infinite cyclic group is isomorphic to (Z,+) and finite cyclic group is isomorphic to (Z,+). Statement of Cayley's Theorem(3).

Natural homomorphism of G onto G/N, N being a normal subgroup of G. First, Second and Third Isomorphism Theorems (3). Isomorphism results relating to normal subgroups(2).

Books Recommended:

(1) Algebra—M. Artin

(2) Topics in Algebra-I.N.Herstein

(3) Topics in Abstract Algebra-M. K. Sen, S. Ghosh, P. Mukhopadhyay

(4) Contemporary Abstract Algebra — Joseph Gallian

(5) First Course in Abstract Algebra-J. B. Fraleigh

SUBSECTION-II

[12 classes]

Invariant subspaces of a matrix: Eigenvectors, eigen-values, Cayley-Hamilton Theorem ,diagonalisation of matrices.(7)

Quadratic forms: Statement of Sylvester's Law & its application, classification, rank, signature, Reduction to canonical form: Application to 2D,3D geometry.(5)

Books Recommended:

(1) Linear Algebra--a Geometric Approach -- S. Kumaresan

(2) Linear Algebra- Freidberg, Insel, Spence

(3) Linear Algebra- K. B. Datta

(4) Vector and Matrices— A. M. Goon

(5) Higher Algebra (Abstract & Linear) — S. K. Mapa

SECTION-II Analysis-III (50 mark s)

36 classes

36 classes

[24 classes]

Course Objective: Learning and application of : 1. properties of continuous functions defined on closed and bounded intervals: uniform continuity. 2. process of finding successive derivatives of given function, 3. expansion of functions in finite and infinite series 3. process of evaluating Indeterminate Forms. 4. process of finding optimum value of real valued functions of real variable.

Continuity of a function: Cauchy Definition and sequential approach. Neighborhood properties. Continuity of elementary functions. Discontinuity of function: Classification of discontinuities. Continuity of composite function. Piecewise continuous function. Statement and proof of properties of continuous functions on closed intervals: boundedness, attainment of bounds, Bolzano's theorem. Intermediate value property & allied results. Continuous function carries closed and bounded interval into closed and bounded interval. Functions continuous on a closed and bounded interval I is uniformly continuous in I. Lipschitz condition and uniform continuity. Statement of existence of inverse function of a strictly monotone function and its continuity (special reference to inverse circular functions).[17]

Introduction to Derivative: Concept of differentiability and differential: Chain rule, sign of derivative. For a differentiable function Lipschitz condition is equivalent to boundedness of the derivative. Successive derivative: Statement (no proof) of Leibnitz theorem. Theorems on derivatives: Darboux theorem. Rolle's theorem, Mean value theorems of Lagrange and Cauchy, Taylor's theorem with Schlomilch & Rouche's form of remainder, Lagrange's and Cauchy's form of remainder. Young's form of Taylor's theorem. Maclaurin series Expansion of e^x , a x^x (a>0), log (1+x), (1+x)^m, sin x, cos x etc. with their ranges of validity. [14]

Indeterminate forms: L. Hospital's rule and its applications. (2)

Maxima/Minima: Point of local extremum (maximum, minimum and saddle point) of a function in an interval. Sufficient condition for the existence of a local maximum/minimum of a function at a point. Application of the principle of maxima-minima in geometrical and physical problems. (3)

Books Recommended:

- (1) Introduction to Real Analysis—Bartle, Sherbert
- (2) Calculus (Vol. I)—T. M. Apostol
- (3) Mathematical Analysis-S. C. Malik and Arora
- (4) Advanced Calculus (An Introduction to Classical Analysis)– Louis Brand (Dover)
- (5) A First Course in Real Analysis—S. K. Berberian
- (6) Basic Real Analysis- Houshang H. Sohrab

PAPER-VI (MT33061)

Elements of Mechanics (100 marks)

(72 classes)

Course Objective: Learning and application of : 1. Newton's laws of motion and principle of conservation of energy: examples from one and two dimensional motions, 2. motion under central force, 3. concept of astatic equilibrium and Poinsot's central axis.

Fundamental Principles of Dynamics (6):

Newton's laws of motion in inertial reference frames, Law of conservation of linear momentum, Work, Power and energy, Work-Energy theorem, Principle of conservation of energy, Energy conservation principle for springmass system.

Motion in One dimension (main stress will be laid on problems) [15]

(a)Simple Harmonic motion- definition, properties, damped oscillation, damped forced oscillati

(b) Motion of a projectile under gravity in a resisting medium-concept of terminal velocity.

- (c) Motion in a straight line under variable acceleration.
- (d)Motion of a particle tied to an elastic string or elastic spring.
- (e) Motion of a connected system.

Motion in Two dimensions (main stress will be laid on problems)[35]

Introducing different co-ordinate systems (Cartesian, Polar, Tangent-Normal), depending on the symmetry of the problems under discussion.-all being viewed as different choices of basis of \square^2 .

Deduction of expressions of velocity & acceleration of a moving particle in Cartesian, Polar, Tangent-normal and rotating co-ordinate system by either vector or matrix method.

Inertial Cartesian co-ordinate system in 2D and motion of a projectile under gravity in a resisting medium-concept of terminal velocity.

Motion of a particle described by plane polar co-ordinate system in 2D.

Central force & central orbits. Central orbit is a plane orbit. Characteristics of a central orbit-Apses & Apsidal angle. Central orbit in a resisting medium.

Constrained motion in 2D-motion of a particle on a rough or smooth plane curve, specially cycloid and parabola and circle. Motion under Inverse square law and classification of conical orbits.

Principle of Varying mass (2)

Forces in 2D: Coplanar forces and Astatic Equillibrium (6)

Forces in 3D: Poinsot's Central Axis and its uniqueness .Wrench, Pitch, Intensity, Invariant of a given system of forces: simple problems. (8)

Books Recommended:

(1) Classical Mechanics—N. C. Rana, P. S. Joag

(2) Dynamics of a Particle and of Rigid Bodies-S. L. Loney

(3) Analytical Dynamics— Saha & Ganguly

(4) Analytical Statics—Sinha & Pradhan.

Paper-VII (100 marks)(MT34071)

Section –I: Introduction to Computer Programming 20 marks

Course objective: Learning and application of : 1.elements of computer software and hardware. 2. elements of C programming.

Computer Fundamentals. Evolution of Computers. Generations of Computer.

Basic Computer Organisation: Input Unit, Output Unit, Storage Unit (Primary and Secondary Storage devices), Arithmetic Logical Unit, Control Unit, Central Processing Unit.

Storing of data in a computer: BIT BYTE, word.

Coding of data: ASCI-I, EBCDIC(Extended Binary Coded Decimal Interchange Code) etc.

Operating System: Basic Idea.[3]

Algorithm and flow chart.[2]

Programming Languages: General Concept ,Machine Language, Assembly Language, High Level Language, Compiler, interpreter. Object and Source Program.[1]

Introduction to C programming Language.

Data types.

Operators, Statements, expression, Input/Output statements.

Control statement: Decision control, loop control.

Array, pointers, stack, queue and function.[4]

Introduction to simple programme: sorting, searching.[2]

Section-II (50 marks) [Numerical Analysis Theory] 36 classes

Course objective: Learning and application of : 1. process of finding value of a function and its derivative where analytical expression of the function is unknown and corresponding error management, 2. process to estimate value of an integral where it cannot be found analytically, 3. process to find solution of an equation or a system of linear equation by different methods and corresponding rate of convergence.

Errors in Numerical Computations : Relative error , Absolute error , Percentage error , round-off rules and Round-off error ,inherent errors , Significant digits and Numerical instability. Error of a sum, difference, product & quotient of two approximate numbers (3)

Operators $\Delta \nabla$, μ , δ , E (Definitions and simple relations among them)(2).

Interpolation: Polynomial Interpolation, Weierstrass Approximation Theorem(statement only). Vandermonde's determinant. Equi-spaced arguments. Difference Table. Different interpolation formulae viewed as various basis choices of the vector space $P_n[a,b]$.Deduction of Newton's Forward and Backward interpolation and Lagrange's interpolation formula and their error estimate. Drawback of Lagrangian Interpolation Formula and preference for divided difference formula. Newton's divided difference formula identified as a discrete version of Taylor's finite series. Inverse Interpolation. (10)

Numerical Integration: Integration of Newton's interpolation formula. Newton-Cotes'formula. (with derivation) Basic Trapezoidal ,Simpson's 1/3 rd, Simpson's rule 3/8 rule and their composite forms. Error estimates of these formulae. Degree of precision (definition only) .(6)

Numerical Solution of non-linear equations: Location of a real root by Tabular method. Bisection method. Regula-Falsi and Newton-Raphson methods, their geometrical significance. Fixed point iteration method. (6)

Numerical solution of a system of linear equations: Direct method— [Gauss elimination method, Operation count. Gauss- Jordan elimination method]. Iterative method—[Jacobi iteration method, Gauss- Seidel method] (6)

Solution of Ordinary Differential equations—Euler's method,Picard method, Runge-Kutta method fourth order). (Single step methods) Multistep methods: Adam's Bashforth method. **(3)**

Books Recommended:

- (1) Introduction to Numerical Analysis Devi Prasad
- (2) Elementary Numerical Analysis Conte de Boor
- (3) Elementary Numerical Analysis Atkinson
- (4) Computational Mathematics-B. P. Demidovich & I. P. Maron

Section-III (Total and Partial Differential Equation) (30 marks)

36 classes

Course Objective: Learning and application of : 1. formation of pde, 2. Processes of finding solution of linear and non-linear pde.

Total differential equations: condition of integrability, methods of solution (3).

Partial Differential Equation(**PDE**) : Introduction, Formation of PDE, Characteristics, Solution of first order linear PDE by Lagrange's Method of solution and non-linear pde by Charpit's Method(**17**).

PAPER- VIII (MT34081)

Section-I :(Algebra -IV) (80marks)

52 classes

Course Objective: Learning and application of : 1. concept of rings and special types of rings, 2. concept of Quotient Ring and Ring Homomorphisms 3. Isomorphism Theorems, 4. definition of PID,ED,UFD and their interrelation.

Rings, Integral Domains, Division Rings, Fields, Sub-rings and Subfields, Basic Theorems(with proof) on Rings, Integral Domains and Fields, Characteristic of a Ring.

Ideals, Quotient Rings and Ring Homomorphisms, Isomorphism Theorems, Chinese Remainder Theorem, Prime Ideals and Maximal Ideals

Embedding of an Integral Domain in a field. Irreducible and Prime elements. Principal Ideal Domain (PID), Euclidean Algorithm and Euclidean Domain(ED). Every ED is a PID, GCD, Unique Factorization Domain (UFD), Every PID is a UFD, Properties of the ring \Box [*i*].

Application to Fermat's two-square theorem.

Books Recommended:

(1) Algebra—M. Artin

- (2) Topics in Abstract Algebra-M. K.Sen, S. Ghosh, P. Mukhopadhyay
- (3) Modern Algebra—S.Singh, Q.Zameruddin

(4) First Course in Abstract Algebra—J.B.Fraleigh

(5) Abstract Algebra—D.S.Dummit, R.M.Foote

Section-II: Number Theory (20 Marks)

20 classes

Course Objective: Learning and application of 1. infiniteness of the set of prime numbers 2. Fundamental Theorem of Arithmetic, 3. Congruence Arithmetic.

Integers: Well-ordering Principle; divisibility and properties of divisibility; prime and composite numbers; infiniteness of the set of prime numbers; Fundamental Theorem of Arithmetic, division algorithm, Euclidean algorithm and expression of g.c.d. as linear combination; Linear Diophantine Equation. Congruence Arithmetic—Fermat's and Wilson's Theorem, Chinese Remainder Theorem, Euler's ϕ function Introduction to Brahmagupta-Pell's Equation.

Note: Adequate emphasis on problem solving to be given.

Books Recommended:

- (1) Elements of Number Theory John Stillwell.
- (2) Introduction to the Theory of Numbers Niven & Zuckerman

(3) Classical Introduction to Modern Number Theory— Ireland & Rosen

(4) A Friendly Introduction to Modern Number Theory—Silverman.

PAPER-IX (MT34091)

Metric Spaces (*with special reference to Real Analysis*) [Approach: Concrete to abstract: Examples, Definition, Theorem, results]

72 classes

Course Objective: Learning and application of : 1. concept of cardinality of a set: denumerable and nondenumerable set: denumerability of Q, non-denumerability of R 2.topology of metric spaces, 3. concept of completeness, compactness and connectedness with reference to metric space and their preservance under homeomorphisms.

Concept of finite and infinite sets, Cardinality of a set, Denumerable, at most denumerable & non-denumerable sets. A subset of a denumerable set is either finite or denumerable. Union of (i) a finite set and a denumerable set (ii) two denumerable sets (iii) denumerable number of denumerable sets. Cartesian product of two denumerable sets. Denumerability of the set of rational numbers, Non-denumerability of points in intervals [a, b]; (a, b); [a, b), (a, b] and the set of all real numbers. (7)

Examples of metric on the set R, the two dimensional plane R^2 &three dimensional space R^3 . Definition and examples of abstract metric spaces: R^n , $l_p (1 \le p)$, discrete metric space, Sup-metric on C[a,b], Minkowski's metric l_p^n , standard bounded metrics, Idea of Pseudo-metric .(6)

Open and closed balls in various metric spaces, its geometries. Interior points, open sets and its examples and basic properties (2), The topology of a metric space, structure of open sets in a metric space, intersection of infinite numbers of open sets may not be open (2). Interior of a set and its basic properties. Limit points, closed sets and its basic properties (1). Closure of a set and its basic properties in a metric space, union of infinite numbers of closed sets may not be closed. Closure of A is the smallest closed set containing A. Dense subsets of a metric space (2). Subspace of a metric space, structure of open and closed sets in subspace (1). Relations of interior and closure of a set in a subspace in comparison with that in the mother space (1).

Bounded sets in a metric space. Diameter of a subset of a metric space and its properties (1). Concept of distance of a sub set from a point, examples, distance between two subsets in a metric spaces its examples and properties, two disjoint closed sub sets may have zero distance (1). Equivalent metric, its examples. Every metric space has an equivalent metric which is bounded (1).

Concept of convergence of a sequence in a metric space with examples, general definition, uniqueness of limit of a sequence in a metric space, bounded sequence, convergent sequence is bounded, bounded sequence may not have a convergent sub sequence (make a tally with the results in usual metric space) (3). Cauchy sequence, Cauchy sequence may not be convergent but it is bounded. Convergent sequence is Cauchy (2).

The concept of completeness in a metric space with examples, definition and non-examples. A subspace in a complete metric space (M, ρ) is complete iff it is closed in (M, ρ) (3).

R with the usual metric is complete and its other equivalent forms and their implications (2). Cantor's Intersection Theorem and its applications (2).

Concept of continuity of functions between metric spaces. Continuity of real valued functions.

Continuity at a point and continuity on a set. Sequential criteria of continuity. Continuity via open sets. Composition of continuous functions is again continuous, algebra of continuous functions. Continuity of vector valued functions (4).

Homeomorphic and isometric spaces—homeomorphism is topological equivalence—isometricism is metrical equivalence. Topological properties, completeness is not a topological property.

Homeomorphic subsets of R, R^2, R^3 (3). The set of discontinuities of a function, the set of discontinuity of a monotone function is at most countable (1).

The concept of Connectedness in a metric space, connected sub sets of real line (2), some special connected sub sets of plane and space (2): Intermediate Value Theorem and its applications. Continuous image of a connected subset of a metric space is connected, connectedness is a topological property. Intersection and union of connected space may not be connected, subspace of connected space may not be connected (2). Classification of subsets of reals, plane and space via connectedness (2).

If $f:[a,b] \rightarrow R$ is continuous and injective then f is strictly monotonic function.

If $f:[a,b] \rightarrow R$ is continuous and assumes every value between f(a), f(b) exactly once in [a,b], then f is strictly monotone on [a,b] (2)

Bounded sequence may not have a convergent subsequence in a metric space, Concept of sequential compactness (1). Bounded infinite subset in a metric space may not have a limit points, the concept of BW-Compactness Equivalence of sequential compactness and BW-compactness (2). Compactness via open cover. Sub space of a compact space may not be compact, Intersection and union of compact spaces. Closed subset in a compact metric space is compact (2). Finite Intersection Property and its use to prove the non-compactness of Euclidean spaces (1) Total boundedness. Proofs of four equivalent statements, viz,

(a) (M, ρ) is compact MS (b) (M, ρ) is sequentially compact MS (c) (M, ρ) is complete MS and totally bounded (d) (M, ρ) has BWP (2).

Compact subsets of the set of reals, plane and space. Compact subsets are closed and bounded in a metric space converse may not be true in general, Heine-Borel theorem.(2) Closed and bounded sets i.e. compact sets in R must have maximum/minimum elements (1).

Continuous image of a compact set is compact. Compactness is a topological property. Real valued continuous function defined on compact domain is bounded and attains its maxima and minima (1). The concept of Uniform continuity of functions, examples, non-examples. Uniform continuous function takes Cauchy sequence in to a Cauchy sequence, uniform continuous function has unique continuous extension on the closure of its domain. The distance function from a fixed set in a metric space is uniformly continuous, disjoint closed subsets have positive distance if one of them is compact. Continuous functions defined on compact domain is uniformly continuous (2). An open continuous bijection on a compact domain is homeomorphism. Let f: [a, b] \rightarrow R be continuous and injective implies f is open that means f⁻¹ is continuous.

Let $f:[a,b] \rightarrow R$ be continuous and strictly monotone increasing with c = f(a), d = f(b). Then f([a,b]) = [c,d], f^1 is strictly monotone increasing on [c,d], and f^1 is continuous (same results for strictly monotone decreasing function).

Classification of subsets of the set of R, R^2, R^3 via compactness (3).

Books Recommended:

(1) Topology of Metric Spaces — S. Kumaresan

(2) Metric Spaces— P.K. Jain & Khalil Ahmed

(3) Metric Spaces—M. N. Mukherjee

(4) Metric Spaces— Satish Shirali & H. L. Vasudeva

(5) Elements of Abstract Analysis—Micheal O Searcoid

(6) Metric Spaces— Victor Bryant.

(7) Introductory Real analysis— M. E.Munroe.

(8) Introduction to Topology and Modern Analysis— G. F. Simmons

(9) Topology: A First Course— J. R. Munkres

PAPER- X (MT34101) 72 classes

Section-I: Inner Product Space (20 Marks) [12 classes]

Course Objective: Learning and application of : 1.introduction of length, distance, orthogonality in a linear space through inner product, 2. process of reduction of a base to an orthonormal base.

Definition of inner product on a linear space, Euclidean and unitary space, examples of inner product spaces(3); Cauchy- Schwarz inequality. Length, distance and orthogonality of vectors in an inner product space(4). Orthogonal and ortho-normal basis. Gram-Schmidt orthogonalisation process (4).

Books Recommended:

- (1) Linear Algebra- K. B. Datta
- (2) Higher Algebra (Abstract & Linear) S. K. Mapa
- (3) Elementary Linear Algebra-Howard Anton, Chris Rorres

Section-II: Linear Programming (80 Marks) [60 classes]

Course Objective: Learning and application of : 1. formulation of LPP from real life problem and its solution. 2. dual nature of LPP and simultaneous solution of primal and dual LPP, 3. process of solving transportation and assignment problem. 4. process of solving game theoretic problem.

Examples of Linear Programming Problems, their standard and canonical form (3), Graphical Solution of L.P.P(3).

Definition and examples of convex sets, Extreme points, Hyper planes and Half spaces, directions and extreme directions of a convex set(3), Representation Theorem of polyhedral sets—bounded and unbounded (statement only)(2).

Simplex Method: Extreme points and Optimality, definition of basic feasible solution (b.f.s.), correspondence between b.f.s. and extreme points(4), algebra of the simplex method, interpretation of entering and leaving the basis, unboundedness (6). The Simplex Algorithm—its finite convergence in the absence of degeneracy (2), the simplex method in tableau format, working out of sums (5). Starting solution and convergence: obtaining

initial b.f.s.—introduction of artificial variables (1). The Charnes' Big-M Method (2). Eliminating artificial variables—the Two-Phase Method (3), Informal discussion on degeneracy, cycling & stalling (2). Duality in LP- Formulation of the Dual, Dual of the dual is primal (3), Primal-dual relationships—the Fundamental Theorem of Duality, Complementary Slackness (4). Transportation and Assignment Problems (10). Introduction to Game Theory (10).

Books Recommended:

- (1) Linear Programming and Network Flows-- M. S. Bazaraa, J.L. Jarvis, H.D. Sherali
- (2) Linear Programming G. Hadley
- (3) Linear Programming– D. J. Bhattacharya
- (4) Operation Research– Hamdy & Taha

PAPER-XI(MT35511)

(Practical Paper in Numerical Analysis through C programming) (72 classes)

Course objective: Learning and application of solving Numerical Analysis problems through writing proper codes/programs in C programing language.

[This particular module aims to be beneficial to the students of Mathematics Honours in terms of applicability of their knowledge in computational science. Computer programming is one such medium to cater to this need and see through the applications of Mathematics by generating suitable codes. Here in this module the problems of Numerical Analysis are taken care of by writing proper codes/programs in a particular programming language. This course is designed for programs written in C language. The course intends to take into account of the following considerations while writing a suitable code for a specific problem]:

- (1) Naming a program properly.
- (2) Stating all the steps with proper description / documentation so that one can understand the logical development of the program.
- (3) While writing a program trying to make it as much user friendly as possible.
- (4) Use of functions as much as one can.
- (5) While running the program, the user should see and understand:
- (a) a brief description of the task/job that the program is doing
- (b) what type of data should be given as input
- (c) clear instruction on how to input the relevant data and the nature of out put
- (d) there should be a menu which clearly states what to do and the user can choose from several options
- (e) if there are other options within a particular option, it should be written accordingly for the user .

Brief description on generating a suitable algorithm for a problem and then incorporating through C language, implementation of algorithm through computer programs, elegant (compact) programs and good(fast) programs, algorithm versus function computable by an algorithm, simulation of an algorithm(computer language).

The following set of problems from Numerical Analysis are to be done on computer using C language:

- Interpolation: Newton's Forward and Backward interpolation (Equidistant nodes) Lagrange's and Newton's divided difference interpolation.
- Numerical Integration: Composite Trapezoidal, Simpson's 1/3- rule, Simpson's 3/8th rule, Weddle's rule and Numerical solution of non-linear equations: Method of Tabulation, Bisection, Regula Falsi, Fixed point iteration, Newton-Raphson Numerical solution of a system of linear equations: (Direct method)Gauss elimination and Gauss Jordan.(Iterative method) Gauss-Jacobi and Gauss- Seidel iteration method. Matrix inversion by Gauss method.
- Power methods for finding the extreme eigenvalues.
- Numerical solution of ordinary differential equation—Euler's method, Picard method, Runge-Kutta method (fourth order). (Single step methods) Multistep methods: Adam's Bashforth method.

PAPER-XII(MT35111)

Section-I (50 marks) Analysis-III

36 classes

Course Objective: Learning and application of 1. Riemann integrability of $f:[a,b] \rightarrow R$ and relevant properties. 2. test of convergence of improper integrals: Beta and Gamma functions. 3. differentiation and integration with respect to the parameter under integral sign.

Riemann Integration for bounded functions: Partition and refinement of partition of an interval. Upper Darboux sum U(P,f) & Lower Darboux sum L(P,f) and associated results. Upper Riemann (Darboux) integral and Lower Riemann (Darboux) integral. Darboux's theorem. Necessary and sufficient condition of R-integrability. Riemann Sum: Alternative definition of integrability. Equivalence of two definitions (statement).Definition of a set of measure zero (or negligible set or zero set) as a set covered by countable number of open intervals sum of whose lengths is arbitrary small). A bounded function on a closed and bounded interval is Riemann integrable function) – (statement only). Problems on Riemann integrability of functions with sets of points of discontinuity having measure zero with special reference to monotone functions, continuous functions, piecewise continuous functions with (i) finite number of points of discontinuities, (ii) infinite number of points of discontinuities having finite number of accumulation points. Integrability of sum, scalar multiple, product, quotient, modulus of Riemann integrable functions, Properties of Riemann integrable functions arising from the above results.

Function defined by definite integral and its properties. Anti-derivative (indefinite integral).

Fundamental theorem of integral calculus. First MVT of integral calculus. Statement of second MVT of integral calculus (both Bonnet's and Weierstrass' form).

Definition of log x (x> 0) as an integral and deduction of simple properties including its range. Definition of e and its simple properties. Theorem on Method of substitution for continuous functions. (21)

Improper Integral: Range of integration, finite or infinite. Necessary and sufficient condition for convergence of improper integral in both cases. Tests of convergence: Comparison and μ -Test. Absolute and non-absolute convergence – Corresponding Tests. Beta and Gamma functions – their convergence and inter-relations. Statement of Abel's and Dirichlet's Tests for convergence of the integral of a product. Uniform convergence of Improper Integral by M – Test .(10)

Definite Integral as a function of a parameter: Differentiation and Integration with respect to the parameter under integral sign – Statements (only) of some relevant theorems and simple problems. **(5)**

Section-II (50 marks)

Analysis-IV (50 marks)

39 classes

Course Objective: Learning and application of : 1. preservance of integrability, boundedness and differentiability under uniform convergence as compared to pointwise convergence : power series as a special case, 2. functions of bounded variation 3. expansion of function in Fourier series.

Sequence of functions (defined on a subset of R): Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Dini's theorem on uniform convergence. Weirstrass'M-test. Boundedness. Repeated limits, Continuity. Integrability and Differentiability of the limit function of a sequence of functions in case of uniform convergence.

Series of functions defined on a set: Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Dini's theorem on uniform convergence. Tests of uniform convergence – Weierstrass' M-test. Statement of Abel's and Dirichlet's test and their applications. Passage to the limit term by term . Sum function: boundedness, continuity, integrability, and differentiability of a series of functions in case of uniform convergence (21)

Power Series (P.S.): Fundamental theorem of Power series. Cauchy Hadamard theorem. Determination of radius of convergence. Uniform and absolute convergence of P.S. Properties of sum function. Abel's limit theorems. Uniqueness of power series having same sum function. Exponential, logarithmic and trigonometric functions defined by Power Series and deduction of their salient properties.

Fourier Series: Trigonometric Series. Fourier co- efficients. A periodic function of bounded variation can be expressed as a Fourier series (Statement only). Statement of Dirichlet's conditions of convergence. Half range series, sine and cosine series. (16)

Function of Bounded variation (BV): Definition of function of BV over [a,b]. Monotone functions are of BV. If f be of BV over [a,b], then it is necessarily bounded in [a,b], Continuous functions may not be of BV over [a,b]. If f: [a,b] \rightarrow R be such that f' is bounded in [a,b], then f is of BV over [a,b]. If f, g be of BV over [a,b] then so are | f | , f ± g, f.g, f/g (with suitable restriction on g in the last case). (5)

PAPER-XIII(MT35121)

Section-I (Probability Theory I)(50 marks) 36 classes

Course objective: Learning and application of : 1. classical and axiomatic definition of probability, 2. Baye's theorem on conditional probability, 3. Probability distribution of random variable, 4. Characteristics of discrete and absolutely continuous distributions, 5. Generating functions.

Introduction to probability theory: Random experiment, sample space, events, examples. Classical Definition of Probability, deductions from classical definition, criticisms or shortcomings of classical definition, statistical regularity, frequency interpretation of probability (3), Kolmogorov's axiomatic development of probability (Kolmogorov's probability axioms), probability space, defining probability function (due to Kolmogorov) on finite, discrete and uncountable sample spaces, properties of probability function, Boole's and Bonferroni's inequalities (3), limit of monotone sequence of events, continuity theorem (1).Conditional probability: definition and examples, representation as a probability space, the multiplication rule of probability, Baye's theorem (2), problems on conditional probability(1). Independence of events: definition of independence of two events, examples, extension to a finite collection of events, pairwise and mutual independence, problems (3).

Compound experiments: Independent trials, Bernoulli trials, Binomial law and its Poisson approximation (2).

Univariate probability distributions: Random variables, definition, induced probability space (2), distribution function and their properties (2), discrete random variables: probability mass function and its properties (1), continuous random variables: probability density function and its properties (1), Examples of discrete and continuous distributions (1), function of a random variable, transformation of one dimensional random variable (discrete and continuous), problems (3).

Expectations and Moments: mean and variance of a univariate distribution (1), expected value of a function of a random variable and its properties (2), raw and central moments, skewness and kurtosis (1), factorial moments, determination of mean, variance, skewness and kurtosis for discrete and continuous distributions (2), quantiles: median as a special case (1).

Generating functions in one dimension: Moment generating function: definition and properties, Characteristic function : definition and properties (2), determination of moments by moment generating function and characteristic functions, computation of moment generating function and characteristic functions of one dimensional distributions (2).

SECTION-II (Function of Several Variables (50marks)

36 classes

Course Objective: Learning and application of 1. differentiation as a linear transformation, 2. expansion of a differentiable function in Taylor's series, 3. constrained optimization of differentiable functions.

Level sets &Graphs (2),Continuity and Limits(3),Linear approximation and Differentiability (6),Geometrical Content: Differentiability and Tangent planes (2), Error estimation (2), Chain Rule, Jacobian, Euler's theorem for Homogenous Functions (6),Directional derivative and Gradient (3),Mean Value Theorem(proof included) (2),Higher order derivatives (2),Taylor's Theorem(3),Local and Global Extrema (3),Lagrange's Multiplier (2).

Books Recommended:

- (1) Undergraduate Analysis (I)& (II)—S. Lang
- (2) Vector Calculus—P. Baxandall & H. Liebeck
- (3) Calculus of Manifolds—Spivak

PAPER- XIV(MT35131) (Optional-1)

Option I- Topology (Teaching Method: From concrete to abstract)

Course Objective: Learning and application of : 1. Topological Space as generalization of metric spaces, 2. Study of compactness, connectedness, completeness, countability and separation properties with reference to topological spaces and their (non-) preservance under continuity and homeomorphism .3. Completeness in metric space, Baire Category Theorem, its various forms.

Concept of topology in a metric space: Motivation. Abstract topological space: examples, definition. Concept of basis of topological spaces. More examples, comparison of topologies, concept of neighbourhood, interior, exterior and boundary, open sets, closed sets, closure of a set. Subspace, product space. Closure and interior in subspace and product space. Kuratowskii's closure axiom.

Concept of continuous function in topological spaces. Example, definition, construction of continuous functions. Pasting Lemma. Topological equivalence, Homeomorphs, homeomorphic subsets of R, R^2, R^3 etc. (special mention on known geometric figures).

Convergence of sequence in a topological space. Non-uniqueness of limits, limit of a convergent sequence is unique in a Hausdorff space. First countability in context of convergence of a sequence and limit point. Heine's continuity criterion. Properties of continuous function in connection of Hausdorff space.

Second countability, separability, Lindeloff properties and their interrelation. They are topological properties but some of them are hereditary and some are productive. Separable metric space is second countable.

Separation axioms: $T_0, T_1, T_2, T_3, T_{3.5}, T_4, T_5$ spaces. Examples. Metric space is T_5 . Relation between T_i spaces(i=0,1,2,3,4,5). Regular and Normal spaces, Urysohn's lemma, Tietze's extension theorem(without proof). Metric space is normal, regular Lindeloff space is second countable.

Completeness in metric space, Baire Category Theorem, its various forms, application on the discontinuities of function.

Compactness : concept and examples, definition , [a,b] is compact in R, closed subsets of a compact space is compact. Compact subsets in R. In T₂ space compact subsets are closed. Product of two compact space is compact, compact subsets of Rⁿ, Heine-Borel theorem. Lebesgue covering lemma. Compactness is a topological property. Compact sets and continuous functions, FIP. In a compact space a family of closed sets having FIP has non-empty intersection. Countable compactness. Frechet compactness, first countability and other concept of compactness.

Concept of connectedness in a topological space, examples, definition, equivalent definition, it is not a hereditary property, intersection of connected sets may not be connected, union of a family of connected sets may not be connected. Closure of a connected set is connected. Connectedness is a topological property. Path connectedness: relation to connectedness. Topologist's sign curve. Examples in R, R^2, R^3 etc. Connected subsets of R. Classification via connectedness.

Reference: Topology: A First Course— J. R. Munkres

Theory of Special Relativity (STR) (Special Paper)-100 Marks [60 classes]

Review of Newtonian Mechanics—Inertial Frames. Speed of light and Pre-Einstein Galilean relativity. Michelson-Morley experiment. Lorenz-Fitzgerald contraction hypothesis. Relative character of space and time. [10]

Postulates of special theory of relativity. Lorentz transformation equations and its geometrical interpretation. [5]

Relative kinematics — composition of parallel velocities. Length contraction. Time dilation. Transformation equations for components of velocity and acceleration of a particle and Lorentz contraction factor. **[8]**

Geometrical representation of space-time—Four dimensional Minkowskian space-time of special relativity. Time-like, light-like and space-like intervals. Null cone. Proper time. World line of a particle. Four vectors and tensors in Minkowskian space-time, Orthogonality properties of Four vectors. Group properties of Lorentz transformations. Subgroups of a Lorentz group. Lorentz Invariance. [20]

Relativistic mechanics—variation of mass with velocity. Equivalence of mass and energy. Transformation equations for mass momentum and energy. Energy-momentum four vector. Relativistic force and Transformation equations for its component. Role of STR in explaining & extending certain consequences of Maxwell's equations describing electromagnetism.[17]

Book Recommended: (1) Classical Mechanics—H. Goldstein

(2) Introduction to The Theory of Relativity—P. G. Bergmann

(3) Introduction to Elementary Particles—D. Griffith

PAPER-XV(MT36141)

SECTION-I (Vector Analysis) (50marks)

36 classes

Course Objective: Learning and application of 1. differentiation of a vector- valued function V(t) with respect to the scalar variable t, 2. understanding physical significance of divergence and curl, 3. concept of line integrals, irrotational vector, conservative force, 4. expression of volume integral in terms of surface integral

and surface integral in terms of line integrals.

Vector differentiation : Differentiation of a vector with respect to a scalar variable: vector functions of one scalar variable, derivative of a vector, Implication of the results $\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$ and $\vec{u} \times \frac{d\vec{u}}{dt} = \vec{0}$. Second

derivative of a vector, derivatives of sums and products. (5)

Concepts of scalar and vector fields, physical significance. Directional Derivative. Gradient. Divergence and curl, Laplacian, Statement and proofs of related vector idetities(10)

Vector integration: Line integrals as integrals of vectors, Circulation, Irrotational vector, Work done, conservative force, Potential orientation. Statements (no verification) of Theorems of Gauss, Green and Stokes. Calculation of area, volume, and surface area, length of plane curve. (20)

SECTION-II (Elements of Mechanics II) (50marks) (36 classes)

Course Objective: Learning and application of 1. motion of rigid body, 2. Principal of virtual work, 3. D'Alembert's Principle.

Two possible motion of a rigid body—translation and rotation. Kinetic energy of a rigid body and Inertia Matrix(2). Diagonalisability of the Inertia Matrix and emergence of the ideas of principal moments and principal axes(2). Momental ellipsoids (1). Determination of moments of inertia of various rigid bodies(1)—Theorems of parallel axes and perpendicular axes (1). Equimomental bodies and related results, determination whether a straight line can be a principal axis at any point in the line (3).

Detailed discussion on principle of virtual work and its converse [Only theoretical discussion]. Deduction of conditions of equilibrium of rigid bodies in 2D and 3D (3).

D'Alembert's Principle and general equations of motion of a rigid body. Independence of motion of centre of inertia and motion relative to the centre of inertia: theory and problems (6). Motion of a rigid body about a fixed axis. Compound Pendulum: theory and problems (6).

Motion of a rigid body in two dimensions: theory and elementary problems. Motion under impulsive forces (3).Principles of conservation of linear momentum, angular momentum and energy under finite forces: theory only(2). Idea of stability and energy test (2).

Book Recommended: 1. Classical Mechanics: J.C. Upadhyaya

2. Classical Mechanics: N.C.Rana, P.S.Joag

3. Classical Mechanics: Goldstein, Safko, Poole

PAPER-XVI(MT36151)

SECTION-I (Algebra -V)(50marks)

Course objective: Learning and application of 1. class equation of a finite group, Cauchy's theorem, Sylow theorems, 2. concept of polynomial rings over a ring, UFD, 3. concept of field extension.

Conjugacy relation, Normaliser, Class equation of a finite group. Centre for group of prime power. Cauchy's Theorem and its applications; converse of Lagrange's Theorem for finite commutative group, Sylow's First Theorem, Sylow p-subgroup, statements of Sylow's Second and Third Theorems. (16)

Polynomial Rings: R is an I D implies R[x] is an I D, R is a field implies R[x] is an ED, D is a UFD implies D[x] is a UFD. Sufficient condition of irreducibility of polynomials—Eisenstein's criterion.(13)

Field Extension : Definition & examples of field extension ; Vector space nature of extended field over the given field; Degree of extensions –finite and infinite extensions and their examples; [L : F] = [L : K] [K : F]; Algebraic and transcendental extension; TH:- "Every finite extension is an algebraic extension"; Primitive set and adjunction; TH:- "A finite extension K over a field F is obtained by adjoining a finite number of elements of K to F"; Simple extension- definition and examples; TH :- "Let a be algebraic over F and p(x) be an irreducible polynomial of degree n over F. Then F(a) is isomorphic to F[x] / < p(x) > and any element of F(a) can be expressed uniquely in the form $c_0 + c_1 a + ... + c_n a^n$; c_1 's TM F."; Fundamental Th. of Field extension.(7)

SECTION-II (Complex Analysis) (50marks) 36 classes

Course objective: Learning and application of 1. comparison of real numbers and complex numbers, 2. convergence of sequence and series of complex numbers, 3. extension of real valued functions of real variable like exponential, logarithmic to corresponding complex valued functions of complex variable, 4. analytic functions and their properties, 5. power series of complex numbers (as compared to that of real numbers).

Field structure of complex numbers, field of complex numbers can not be totally ordered, Geometric Interpretation of complex numbers , Topology of the complex plane. Stereographic Projection.

Sequence and series of complex numbers: Notion of Convergence. $\{z_n\}_n$ is convergent iff $\{R_e z_n\}_n$ and $\{I_m z_n\}_n$ both are convergent. Cauchy Condition for convergence. Subsequence in C: every bounded sequence has a convergent subsequence.

Analogous results for convergence of $\sum z_n$. Absolute convergence. Statement of (i) Ratio test, (ii) Root test (iii) Dirichlet's test (iv) Abel's test.

Function of a complex variable – Exponential, Logarithmic, Direct and Inverse Circular and Hyperbolic functions. Injective and Surjective functions, Concepts of limit and continuity, sequential continuity is equivalent to continuity. Continuity and Connectedness. Continuous function on a compact set is uniformly continuous.

Analytic Function: Differentiability – definition, derivability implies continuity, differentiability of sum, difference, product, quotients and composition of differentiable functions, Cauchy – Riemann equations are necessary but not sufficient conditions for differentiability of a function at a point in its domain of definition, sufficient conditions for differentiability. Definition of analytic and entire function. Composition of analytic function.

Harmonic function and Harmonic conjugate : basic results regarding existence and determination of harmonic conjugate. The real and imaginary parts of an analytic function defined on an open subset O of the complex plane are harmonic on O. Milne – Thomson method.

Sequence of functions and series of functions in C: Pointwise and uniform convergence. Cauchy Criterion. M-Test. Statement of results regarding continuity, analyticity of limit function/sum function in case of uniformly convergent sequence/series of function in C.

Power Series as an analytic function: radius of convergence of a power series. (Cauchy-Hadamard Form and Ratio Form) Absolute and uniform convergence of a power series strictly within the circle of convergence. A power series and its derived power series have same radius of convergence. A power series is an analytic function strictly within its circule of convergence (statement only) and conversely if f is analytic in a domain D, then f can be represented by a power series locally about each point z_0 in D. (statement only).

PAPER- XVII(MT 36161) (Optional-2)

• Probability Theory II and Statistics(100 marks)

Section-I Probability theory II (50 marks)

Probability distribution in two dimension: two dimensional random variable, definition and examples (2), joint distribution function, definition and properties, marginal distributions (2), discrete and continuous random variables, joint probability mass function and joint probability density function, definition and properties (2), bivariate normal and uniform distributions (1), conditional distributions: conditional distribution functions for discrete and continuous random variables (1), stochastic independence, transformation of a two dimensional random variable, problems (3).

Two dimensional expectation: expectation of functions of two dimensional random variables, properties, multiplication rule for expectations (2), moments, covariance and correlation co-efficient: definition and its properties (3), conditional expectation and regression (2), quadratic and linear regression curves by least square method, measure of goodness of fit, problems (2).

Generating functions in two dimensions: joint moment generating function, two dimensional moments from moment generating function, joint characteristic function (2), bi-variate normal distribution: determination of moment generating function and characteristic function, marginal and conditional densities (1).

Convergence of sequence of random variables and limit theorems: convergence in probability, Tchebycheff's inequality (3), convergence in mean square, convergence in distribution (2), law of large numbers (2), asymptotic distribution: Limit theorem for characteristic functions, Central limit theorem for equal components (Lindeberg-Levy Theorem), De- Moivre-Laplace Limit theorem (4).

Section II Statistics (50 marks)

Random sampling, sample statistics (2), sampling distributions of certain sample characteristics (sample mean and sample variance), moments of sample characteristics (2), exact sampling distributions: Chi-square, t-, and F-distributions (3).

(69 classes)

Point estimation of a population characteristic or parameter, unbiased and consistent estimates (4), sample characteristics as estimates of the corresponding population characteristics (2), maximum likelihood estimates, application to Binomial, Poisson and Normal populations (3).

Interval estimation: Confidence intervals or confidence limits for mean and standard deviation of a normal population (2). Approximate confidence limits for the parameter of a binomial population (1).

Bivariate samples: scatter diagram, sample correlation co-efficient (1), least square regression lines and parabolas (2).

Statistical hypothesis, simple and composite hypothesis, critical region of a test, type-I and type-II error, power function of a test, best critical region of a test (4), Neyman- Pearson theorem (1) and its application to Normal population (2), likelihood ratio testing and its application to Normal population (tests on mean and standard deviation of a normal population, comparison of means and standard deviations of two normal populations) (4), approximate tests on the parameter of a binomial population (1), on comparison of two binomial populations (1), simple problems of hypothesis testing (1).

• ELEMENTARY DIFFERENTIAL GEOMETRY

100 marks

Curves in R² and R³. Arc length, Reparametrization. Level curves versus Parametrised curves. Curvature and torsion for space curves, Existence theorem for space curves. Serret-Frenet formula for space curves. Inverse and implicit function theorems. Jacobian theorem. Surfaces in R³, regular surface. Inverse image of a regular value change of parameters. Differentiable functions on surfaces. Tangent Plane, the derivative of a map. First fundamental form. Area, orientation of a surface. Characterisation of compact oriented surface. Geometric definition of area.

The geometry of the Gauss maps. Gauss map and its fundamental properties. Second fundamental form, Mean curvature and scalar curvature.

Vector fields. Integral curves of a vector field, existence and uniqueness. Curvature of curves on a surface. Normal and Principal Curvatures. Geometric interpretation of principal curvature. Gaussian curvature and the Gauss maps. Intrinsic geometry of surface. Isometric, conformal maps, the Gauss theorem and equation of compatibility – Parallel transport and Geodesic.

Geodesic equation, Geodesic on surface of revolutions, Geodesic as shortest paths, Geodesic coordinates, Ruled surface and Minimal surface, Gauss Bonnet theorem and its applications.

Reference: 1. Elementary Topics in Differential Geometry—John A. Thorpe

2.Elementary Differential Geometry—Manfreds P. do Coumo

3. Elementary Differential Geometry—Andrew Pressley

• Functional Analysis

Course objective: Learning and application of 1.comparison between (a) linear space and normed linear space and (b) finite and infinite dimensional n.l.s., 2. Criteria of closed and bounded subsets of an n.l.s. to be compact in terms of dimensionality of the space, 3. equivalent criteria of continuity of linear transformations, 4. extension of linear functional, 5. Open Mapping and Closed Graph Theorems.

Vector Spaces, Hamel basis, Infinite dimensional linear spaces, Basis of a linear space exists. Normed linear spaces (n.l.s.), examples of n.l.s. related to finite dimensional spaces, sequence spaces, space of continuous functions. The metric induced by norm. Convergence of sequence in n.l.s. and their properties, closure of a linear subspace is again a linear subspace. Banach space, examples and non-examples, complete subspaces and closed subspaces and their relations, completion of an n.l.s. Finite dimensional subspaces are closed but converse may not be true. Banach space cannot have a countably infinite basis. Separability of n.l.s., Equivalent norms, examples and non-examples, all norms are equivalent on a finite dimensional space. Concept of convergent series in an n.l.s., an n.l.s. is complete if and only if every absolutely convergent series is convergent. Riesz lemma, closed and bounded subsets of an n.l.s. are compact if and only if the space is finite dimensional.

Linear transformations, continuous linear transformations between n.l.s., examples and nonexamples. Equivalent criterion of continuity of linear transformation. Bounded linear transformations, A linear transformation is continuous if and only if it is bounded. Linear functional, space of bounded linear transformations B(X,Y) as an n.l.s., unbounded linear transformations, examples , non-examples. On finite dimensional n.l.s. all linear transformations are bounded, computation of norm of linear functional, Y is complete if and only if B(X,Y) is complete. Dual space.

Extension of linear functional, Hahn-Banach theorem, consequence of Hahn-Banach theorem. Bounded Inverse theorem, open mapping theorem and its applications, closed linear transformations, examples, non-examples. A linear transformation is closed if and only if its graph is closed, closed graph theorem, uniform boundedness principle, Banach-Steinhaus theorem and its applications.

Reference: Introductory Functional Analysis with Applications: E. Kreyszig

PAPER-XVIII(MT36523)

Dissertation /Project* (*Project Report +Seminar Presentation*) & Grand Viva**

*[Dissertation/Project will probably begin in 4th Semester but will include neither Attendance component nor Internal Assessment component]

** [Grand Viva will be on the whole syllabi studied]

DETAILED SYLLABUS OF MATHEMATICS ANCILLARY

Total 300 marks-3 papers, each carrying 100 marks and duration 3hoursFor each 100 marks paper, six classes available per week.For each semester, 12/13 weeks available for class hours

Elective Paper- I (100 marks) (MT 21011)

MODULE –I (Algebra-1) (36 classes) (50 marks)

Course Objective: Learning and application of :1. finding simplified expression of z^n , z complex, n rational, 2. finding or locating roots of a polynomial equation, 3. notions of equivalence relation and partition and their interdependence, 4. mapping: injective, surjective, internal binary operation.

Complex Numbers[4] :De Moivre's theorem and its applications(4)

Polynomials[16]: Fundamental Theorem of Classical Algebra (statement only).Nature of roots of an equation (surd or complex roots occur in pairs) (2).Synthetic Division(2)Statement of Descarte' s rule of signs and its applications(2). Statement of Bolzano's theorem on continuity in case of polynomials. Relation between roots and coefficients(2). Symmetric functions of roots(1). Transformation of equations (4). Cardan' s method of solving a cubic & Ferrari's method for a biquadratic equation(3).

Set Theory& Relations [8]:Laws of algebra of sets &De Morgan's laws(2). Cartesian product of sets(2) Relations on a set. Reflexive, symmetric and transitive properties of a relation on a set(2). Equivalence relations, equivalence class& partitions- illustrative discussions (2).

Mappings[8]: Injective and surjective mapping(2). Composition of mappings—concept only(1). Identity and inverse mappings(2). Binary operations on a set, Identity element & Inverse elements(3)

MODULE -- II (Calculus-1) (39 classes) (50 marks)

Course objective: Learning and application of : 1. Real number system and its completeness in particular, 2. concept of convergence sequence of real numbers, MCT and Cauchy's General Peinciple, in particular, 3. convergence of series of real numbers and related tests, 4. properties of continuous functions defined on closed

bounded interval, 5. process of finding successive derivative, 6. convergence of improper integrals and related tests: Beta and Gamma function.

Real numbers [4]: Axiomatic definition and Cantor's geometric presentation(2), Bounded sets of real numberstheir sup. and inf., Least upper bound axiom(2)

Sequence[10]: Definition, Bounded& unbounded sequence, Monotone sequence(2) Limit of a sequence & its uniqueness .statement of limit theorems(3). Concept of convergence and divergence of monotonic sequences— Statement of Monotone Convergence Theorem and its applications -definition of e. (3) Statement of Cauchy's General Principle of convergence and its applications(2).

Series of constant terms[7]: Convergence and divergence (definitions). Necessary condition of convergence, Geometric & p-series. Cauchy's principle as applied to infinite series (application only)(3). Series of positive terms: Statements of Comparison Test, Ratio Test, Cauchy's Root Test. Raabe 's test—applications(3). Alternating series: statement of Leibnitz Test and its applications(1).

Real valued functions[7]: limit of a function ($\varepsilon - \delta$ definition and Cauchy's definition) and algebra of limits(3). Continuity of a function at a point and in an interval. Acquaintance (no proof) with the important properties of continuous functions on closed intervals(3). Statement of existence of inverse function of a strictly monotone function and its continuity(1).

Derivative[6]:LHD&RHD, Sign of derivative—monotone increasing and decreasing functions. Relations between continuity and derivability(3). Successive derivatives—Leibnitz Theorem and its applications(3)

Improper integrals[5]: Definition, statement of μ -test and comparison tests-simple applications only(3). Use of Beta and Gamma functions (convergence and useful relations being assumed) (2)

Elective Paper- II (100 marks) (MT 22021)

MODULE -I(Algebra-2) (39 classes) (50 marks)

Course objective: Learning and application of : 1.concept of matrix and determinant over real or complex numbers of arbitrary finite order, in particular, symmetric and skew-symmetric matrices, Orthogonal, Unitary & Hermitian matrices, 2. evaluation of determinant (Laplace'smethod), 3. consistency and solution of system of linear equations, 4. definition, examples, elementary properties of group and subgroup, 4. concept of external binary operation, vector space, linear dependence and independence of finite set of vectors, basis and dimension, 5. Cayley-Hamilton Theorem, 6. process of reduction of real quadratic form to its normal form.

Matrices[5]: Algebra of matrices. Transpose of a matrix—its properties. Trace of a square matrix –its basic results. Symmetric and skew-symmetric matrices, Orthogonal, Unitary & Hermitian matrices.

Determinants[8]: properties, cofactors and minors. Product of two determinants(1). Adjoint, Symmetric and skew-symmetric determinants & its properties(2). Laplace's expansion method for fourth order determinants(1). Inverse of a non-singular square matrix by method of adjoint & method of partition(4).

Rank of a matrix: determination of rank either by considering minors or by sweep-out process. Consistency and solution of a system of linear equations with not more than three variables by matrix method, Cramer's rule.[4]

Introduction to Group Theory[6]—definition and examples taken from number system ,roots of unity, 2x2 real matrices, non-singular real matrices of fixed order(3). Elementary properties of groups (1). Definition & examples of a subgroup—statement of necessary and sufficient condition of being a subgroup— its applications(2).

Elementary ideas of Ring & Field as prerequisite to Vector space [2]

Vector Space[7] :Concept of vector space over a field—examples(2), concepts of linear span, linear dependence and independence of a finite set of vectors, subspace(2), Idea of basis of a finite dimensional vector space. Problems on formation of a basis of a vector space (no proof required)(3)

Theory of Eigenvalues[7]: Characteristic equation of a square matrix of order not more than three determination of eigen values and eigenvectors—problems only(3). Algebraic & Geometric multiplicity of eigen values(1). Statement and illustration of Cayley-Hamilton Theorem(1).Real Quadratic forms(2).

MODULE –II (Calculus-2) (39 classes) (50 marks)

Course objective: Learning and application of : 1. Rolle's and MVTs (Lagrange and Cauchy) and their applications, 2. Taylor's and Maclaurin's finite and infinite series expansion of elementary functions and their use in determining optimum value of real valued function of real variable, 3. Limit, continuity, partial derivative and differeniability of function of two real independent variables, 4. Euler's theorem on homogeneous differentiable functions, 5. Process of finding optimum of function of not more than three independent variables, 6. Constrained optimization, 6. Formation of ODE, 7. First order first degree ode: exact and IF for non-exact, linear and Bernoulli 7. First order higher degree ode and singular solution, 8. Second order linear ode: method of variation of parameters.

Mean Value Theorems[10]:Statement of Rolle's Theorem - its geometrical interpretation and direct applications .Mean Value Theorems of Lagrange and Cauchy(proof included) and applications(3) . Indeterminate Forms: L' Hospital's Rule: statement and problems only (2).

Statement of Taylor's and Maclaurin's Theorem with Lagrange's & Cauchy's form of remainders. Taylor's and Maclaurin's infinite series for functions like $\exp(x), \sin(x), \cos(x), (1+x)^n, \ln(1+x)$ (with restrictions whereever necessary)(3). Application of the principle of maximum and minimum for a function of a single variable in geometrical, physical and other problems(2).

Functions of two and three variables 10]: Geometrical representations. Limit and continuity (definitions) for functions of two variables(2). Partial derivative: knowledge and use of chain rule. Exact differentials (emphasis on problem solving only)(2). Functions of two variables—successive partial derivatives: statement of Schwarz's theorem on commutativity of mixed derivatives(1). Euler's Theorem on homogeneous function of two and three variables (2). Maxima and minima of functions of not more than three variables—Lagrange's method of undetermined multiplier—problems only(using theory of eigen values)(3). Working knowledge of Double Integrals [3]. -Problems only

working knowledge of Double Integrals [0]. Troblems only

Ordinary Differential Equation (O.D.E.)[16] Formation of ode -exemplification from various fields(2)

First order ode : Exact differential equations, Non-exact differential equations & Integrating factors(no proof)(4).Linear ode and Bernoulli's equation(2),Clairaut's equation: general& singular solutions(2).

Second order linear differential equations with constant co-efficients: D operator, Homogeneous and non homogeneous ode (Matrix method to be encouraged)(4).Method of variation of parameters for solving inhomogenous linear ode(2).

Elective Paper –III (Students may opt for any two of the three modules) (100 marks)(MT23031)

Module-I [Numerical Analysis (*Optional Module*)] (36 classes) (50 marks)

Course objective: Learning and application of : 1. theory of errors and $operators \Delta$, ∇ , E, 2. process of finding value of a function where analytical expression of the function is unknown and corresponding error management,3. numerical pifferentiation for equi-spaced arguments,4. process to estimate value of an integral where it cannot be found analytically,3. process to find solution of an equation or a system of linear equation by different methods.

Preliminaries: Approximate numbers, Significant figures, Rounding off numbers. Theory of Errors(2) Operators-- Δ , ∇ , E(definitions and some relations among them) (2)

Interpolation: the problem of interpolation. Equi-spaced arguments—difference tables(2), Deduction of Newton's Forward Interpolation Formula, Remainder Term (proof included)(2). Deduction of Newton's Backward Interpolation Formula, Remainder Term(proof included)(1). Lagrange's Interpolation Formula for unequally spaced arguments—(statement only). Numerical problems on interpolation with both equal and unequally spaced arguments(3).

Numerical Differentiation for equi-spaced arguments(1).

Numerical Integration: Trapezoidal and Simpson's 1/3 rd formula, Weddle's rule (Simple derivations and .geometrical significance(5)). Numerical problems(3).

Solution of Equations: Finding real root of an algebraic or transcendental equation.

Location of root (tabular method), bisection method, Secant method(5), Newton-Raphson method with geometrical significance,-Numerical problems(3)

Solution of a system of linear equations: Gauss Elimination and Gauss-Seidel methods(6)

Module-II [Elementary Operation Research (*Optional Module*)] (36 classes) (50 marks)

Course Objective: Learning and application of : 1. formulation of LPP from real life problem and its solution. 2. dual nature of LPP. 3. Process of solving transportation and assignment problem. 4. Basic inventory controls model.

Motivation of Linear Programming Problem. Statement and formulation of L.P.P.(3) Solution by graphical method (for two variables)(2),Convex set, hyperplane, extreme points, convex polyhedron(3), basic solutions and basic feasible solutions (b.f.s.). Degenerate and non-degenerate b.f.s.(2).

The set of all feasible solutions of an L.P.P. is a convex set. The objective function of an L.P.P. assumes its optimal value at an extreme point of the convex set of feasible solutions. A b.f.s. to an L.P.P. corresponds to an extreme point of the convex set of all feasible solutions(3).

Fundamental Theorem of L.P.P.(statement only)(1). Reduction of a feasible solution to a b.f.s.(1) Standard form of an L.P.P. Solution by simplex method (3) and method of penalty & Two Phase method(3). Duality theory-The

dual of the dual is the primal, relation between the objective values of dual and the primal problems(2). Dual problems with at most one unrestricted variable and one constraint of equality(2).

Transportation (4) and Assignment problem(4) and their optimal solutions. Inventory Control(2).

Module-III [Advanced Calculus] (*Optional Module*)] (36 classes) (50 marks)

Course objective: Learning and application of : 1. properties of sum function of a power series, 2. Fourier series expansion of functions, 3. formation and solution of first order pde: Lagrange's and Charpit's method,4. Frobenius method of series solution of a second order ode at a regular singular point, 5. use of Laplace transform in solvive linear ode.

Power series[8]: Idea of sequence & series of functions. Power series and its radius of convergence. Statement of properties of continuity of sum function of power series. Term by term integration and differentiation of power series. Statement of Abel's theorem on power series. Simple problems.

Fourier series[4]-Statement (no proof) of basic theory ,Dirichlet conditions- simple problems

Partial differential equation (PDE) [8] : Introduction, Lagrange's method for solving linear pde of 1^{st} order, Charpit's method for nonlinear pde of 1^{st} order. Second order pde : Classification & simple examples.

Series solution of second order ode[6]: Idea of ordinary points & singular points of an ode. Power series method and Frobenius method of series solution of ode .

Special functions [6]: Legendre Polynomials, Bessel functions, Airy functions.

Laplace Transform [4]: Elementary idea and use of it to solve ode.

(1) Introduction to Real Analysis—Bartle, Sherbert

- (2) Calculus(Vol.I)—T.M.Apostol
- (3) Undergraduate Analysis—S.Lang
- (4) Mathematical Analysis—Shanti Narayan
- (5) Differential Calculus Shanti Narayan
- (6) Integral Calculus—Shanti Narayan
- (7) A First Course in Real Analysis—S.K.Berberian
- (8) Theory and Applications of Infinite Series—K.Knopp
