

Semester-1

Core Paper 2: **Foundation in Algebra**

Paper Code: HMTCR1021T, Full Marks: 100 (78 Classes), Total Credit:6=5+1(Th+Tutorial)

Course objective: Familiarize the students with methods of solving algebraic equations. Acquainting students with salient properties of integers. Helps students to understand the role of matrix in solve system of linear equations and introduces students the linear structure.

Module-I: Classical and Abstract Algebra (39 Classes)

Unit 1

Modulus and Amplitude (principal and general values) of a complex number: Polar representation **(3)**, De Moivre's theorem for rational indices and its applications: nth roots of unity. **(3)**

Theory of equations: Fundamental Theorem of Classical Algebra(statement only) Relation between roots and coefficients **(3)**, transformation of equation **(2)**, Descartes rule of signs **(1)**, solution of cubic and biquadratic equations: Cardan's and Ferrari's method. **(4)**

Inequality: The inequality involving $AM \geq GM \geq HM$, Cauchy-Schwartz inequality. **(4)**

Unit 2

Binary Relations. Equivalence relation and Partition: Their Equivalence **(4)** Functions: Injective, Surjective, Bijjective. Composition of functions, Invertible functions **(5)**. Well-ordering property of positive integers, division algorithm, divisibility and Euclidean algorithm (proofs required) **(5)**. Congruence relation between integers. **(2)** Principles of Mathematical induction, Statement of Fundamental Theorem of Arithmetic. **(3)**

Module-II: Matrix Algebra (39 Classes)

Unit 1

Systems of linear equations $Ax=b$: Homogenous and Non-Homogenous systems **(2)**. Elementary Row Operation: row reduction and echelon forms, rank of a matrix **(7)**. Characterization of invertible matrices using rank. Consistency of a system of linear equations **(4)**. Solution set of linear systems: Gauss Elimination method and Matrix Inversion Method. **(4)**

Unit 2

\mathbb{R}^n over \mathbb{R} as a vector space and its subspaces **(4)**. Introduction to linear transformations as structure preserving maps, matrix of a linear transformation **(6)**. Solution space of a system of homogenous linear equations is a subspace **(2)**, Linear Independence and Spanning Subsets in \mathbb{R}^n . Dimension of subspaces of \mathbb{R}^n . **(5)** Eigen values, eigen vectors and characteristic equation of a matrix. Cayley-Hamilton theorem and its use in finding the inverse of an invertible matrix **(5)**.

References:

1. **The Theory of Equations Vol.1 and Vol.2: Burnside and Panton.**
2. Higher Algebra—Barnard and Child.
3. Higher Algebra (Classical, Abstract& Linear) — S. K. Mapa
4. Modern Algebra—Surjeet Singh and QaziZameruddin
5. Linear Algebra-K. Hoffman and R. Kunze.
6. Topics in Algebra—I. N. Herstein
7. Topics in Abstract Algebra—M. K. Sen, S. Ghosh, P. Mukhopadhyay
8. Elementary Linear Algebra—Howard Anton, Chris Rorres
9. Linear Algebra- K. B. Datta