## Core 4.1: Probability Theory and Multivariate Calculus.

**Theory Paper, Full Marks: 100, Total Credit:6=5+1(Th+Tutorial), No. of classes per week:6(=5+1)**
**Paper Code: HMTCR4081T**

## Module-I: Probability Theory (39 classes)

**Course Objective:** *The aim of this course is to give a proper mathematical treatment to the word 'Probability' which is a well-known phrase used by almost all of us. To give students an acquaintance with the axiomatic development of probability theory by A.N.Kolmogrov and develop a mathematical theory with the help of induced probability space and distribution functions.*

Experiments: Deterministic and Non-deterministic. Interpretation of randomness in non-deterministic experiments. Examples from different fields.

Sample space connected to different random experiments, examples [finite, countably infinite and uncountable]. [2]

Events. Elementary and compound events, examples. Formation of new events through different algebraic operations on them [union, intersection, complement].[1] Definitions of (1) sure event (2) impossible event (3) mutually exclusive events along with examples.[1]

Analogy to set theory. Class of sets. Field, sigma-field , examples[1]. Idea of minimal sigma-field. Partition. Construction of minimal sigma field. Borel field, Borel sets in R.[2] Sequence of sets/events. Monotone sequence of events: contracting and expanding sequence. Limit of monotone sequence of events. Limit superior and limit inferior of an arbritary sequence of events.[3]

Idea of pair-wise disjoint /mutually exclusive, mutually exhaustive events for a class of events, examples. Given a sequence of events there exists a pair-wise mutually exclusive sequence of events such that their unions are equal [Proof included].[2]

Introduction to the idea of probability: different interpretations: (1) Frequency interpretation (2) Classical interpretation [criticism or shortcomings of this approach, problems][2] (3)Kolmogorov's Axiomatic approach[ Kolmogorov's probability axioms].[1]

Probability space. Probability function defined on finite, countably infinite and uncountable sample spaces. Properties of probability function.[3] Boole's and Bonferroni's inequality[ Proof included]. Continuity theorem.[3]

Conditional Probability. definition ,examples, Representation as a probability space.[2] multiplication rule of probability, Baye's theorem , problems. [2]

Independence of two events. extension to a finite/ countably infinite collection of events, pairwise and mutual independence, problems.[2]

Trials. Independent trials[ Bernoulli][1]

Introduction to random variables: real valued function and inverse set function, inverse mapping preserves all set theoretic relations.[1]

Random variable as Borel measurable function. Indicator function. Simple function. Induced probability space [justification as probability space according to Kolmogorov's axioms]. [2]

Distribution function. Properties.[2]Classification of random variables: discrete and absolutely continuous random variables. Probability mass function and probability density function.[1]Transformation of one dimensional random variable (discrete and

continuous), problems.[2]

Examples of Discrete and Continuous random variables: Binomial, Poission [Discrete], Uniform, Normal [continuous].[3]

References:

1. Basic Probability Theory: Robert B-Ash
2. Introduction to Probability Theory: Feller Vol.1
3. Introduction to Probability Theory: Sheldon Ross
4. Modern Probability Theory: B.R.Bhatt
5. Introduction to Probability Theory: Parzen
6. Mathematical Probability: Banerjee, De, Sen

## Module-II : Multivariate Calculus (39 classes)

**Course Objective:** *To learn the continuity and differentiability of functions of more than one variable, derivative as a linear map, the role of gradient of a function and related geometry, chain rule, MVT, Inverse and Implicit function theorem and their applications (geometric), extreme values of functions and conditional extrema.*

Concept of limit, Continuity of functions of two real variables, continuity of vector valued functions (5). Partial derivatives and directional derivatives and its relation with continuity, sufficient conditions of continuity (5). Differentiability of two variables functions and its relation with partial derivatives and continuity, sufficient conditions of differentiability, commutativity of mixed order partial derivatives (5). Differentiability of vector valued functions, examples, linear maps , bilinear maps, differentiability of standard maps, jacobian matrix and chain rule (5). Mean value theorem of scalar valued maps on vector domains, Mean value inequality of vector valued maps, functions of vanishing partial derivatives on connected domains (4).

Functions with non-singular derivatives, Inverse function theorem, Implicit Function theorem and their applications (5). Level surface, tangent space to the regular level surfaces, gradient as normal to the regular level surfaces, Critical points, Extreme values, Lagrange's multiplier method, local expression of a function near non-degenerate critical points (10).

References:

1. Calculus: T. M. Apostol vol.II
2. Calculus on Manifolds: M. Spivak
3. Multivariate Calculus and Geometry: Sean Dineen
4. Basic Multivariate Calculus: A. Weienstein, J. Marsden, A. Tromba