C4.2: Riemann Integration and Metric Spaces-I

Theory Paper, Full Marks: 100, Total Credit:6=5+1(Th+Tutorial), No. of classes per week:6(=5+1)
Paper Code: HMTCR4091T

Module-I: Riemann Integration (39 classes)

Course Objective: To learn

1. the theory and applications of Riemann Integration of a bounded real valued functions defined on a closed and bounded interval.
2. The improper integration, Beta and Gamma integrals.

Riemann Integration for bounded functions: Partition and refinement of partition of an interval. Upper Darboux sum U(P,f) & Lower Darboux sum L(P,f) and associated results. Upper Riemann (Darboux) integral and Lower Riemann (Darboux) integral(**3**). Darboux's theorem. Necessary and sufficient condition of R-integrability (**2**). Riemann Sum: Alternative definition of integrability. Equivalence of two definitions (statement)(**2**).Definition of a set of measure zero (or negligible set or zero set) as a set covered by countable number of open intervals sum of whose lengths is arbitrary small).A bounded function on a closed and bounded interval is Riemann integrable if the set of points of discontinuity is a set of measure zero (Lebesgue's theorem on Riemann integrable function) (**3**). Problems on Riemann integrability of functions with sets of points of discontinuity having measure zero with special reference to monotone functions, continuous functions, piecewise continuous functions with (i) finite number of points of discontinuities, (ii) infinite number of points of discontinuities having finite number of accumulation points(**5**). Integrability of sum, scalar multiple, product, quotient, modulus of Riemann integrable functions, Properties of Riemann integrable functions arising from the above results(**2**).Function defined by definite integral and its properties. Anti-derivative (indefinite integral).Fundamental theorem of integral calculus and its consequences (**3**). First MVT of integral calculus, second MVT of integral calculus (both Bonnet's and Weierstrass' form) and their applications (**5**).

Definition of log x (x> 0) as an integral and deduction of simple properties including its range (**2**). Definition of e and its simple properties.Theorem on Method of substitution for continuous functions (**2**). **[29]**

Improper integrals their convergence, μ-test, comparison test and their applications. Convergence of Beta and Gamma functions. **[10]**

Reference:

- K.A. Ross, Elementary Analysis, The Theory of Calculus, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.
- R.G. Bartle D.R. Sherbert, Introduction to Real Analysis, 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.
- Charles G. Denlinger, Elements of Real Analysis, Jones & Bartlett (Student Edition), 2011.
- S. Goldberg, Calculus and mathematical analysis.
- Santi Narayan, Integral calculus.
- T. Apostol, Calculus I, II.

Module-II: Metric Spaces-I (39 classes)

Course Objective: Learning and application of : 1.topology of metric spaces, 2. concept of convergence of a sequence and completeness,

Examples of metric on the set R , the two dimensional plane $\mathbb{R}^2$ &three dimensional space . $\mathbb{R}^3$

Definition and examples of abstract metric spaces: $\mathbb{R}^n$ l discrete metric space, Sup- metric on $C[a,b]$, Minkowski's inequality. l^p **space.** standard bounded metrics, Idea of Pseudo-metric .**(6)**

Open and closed balls in various metric spaces, its geometries. Interior points, open sets and its examples and basic properties (**2**), The topology of a metric space, structure of open sets in a metric space, intersection of infinite numbers of open sets may not be open (**2**). Interior of a set and its basic properties. Limit points, closed sets and its basic properties (**2**). Closure of a set and its basic properties in a metric space, union of infinite numbers of closedsets may not be closed. Closure of A is the smallest closed set containing A. Dense subsets of a metric space (**2**).

Subspace of a metric space, structure of open and closed sets in subspace (**1**). Relations of interior and closure of a set in a subspace in comparison with that in the mother space (**1**).

Bounded sets in a metric space. Diameter of a subset of a metric space and its properties (**1**). Concept of distanc of a sub set from a point, examples, distance between two subsets in a metric spaces its examples and properties,two disjoint closed sub sets may have zero distance (**3**). Equivalent metric, its examples. Every metric space has an equivalent metric which is bounded (**3**).Concept of convergence of a sequence in a metric space with examples, general definition, uniqueness of limit of a sequence in a metric space, bounded sequence, convergent sequence is bounded, bounded sequence may not have a convergent sub sequence (make a tally with the results in usual metric space) (**4**). Cauchy sequence, Cauchy sequence may not be convergent but it is bounded. Convergent sequence is Cauchy (**2**).

The concept of completeness in a metric space with examples, definition and non-examples.A subspace in a complete metric space (M, ρ) is complete iff it is closed in (M, ρ) (**4**).

R with the usual metric is complete and its other equivalent forms and their implications (**3**). Cantor's Intersection Theorem and its applications (**3**).

Books Recommended:

(1) Topology of Metric Spaces — S. Kumaresan
(2) Metric Spaces— P.K. Jain & Khalil Ahmed
(3) Metric Spaces—M. N. Mukherjee
(4) Metric Spaces— Satish Shirali & H. L. Vasudeva
(5) Elements of Abstract Analysis—Micheal O Searcoid
(6) Metric Spaces— Victor Bryant.
(7) Introductory Real analysis— M. E. Munroe.
(8) Introduction to Topology and Modern Analysis— G. F. Simmons