Semester-2
GE 2:**Advanced Algebra and Calculus**

Paper Code: HMTGE2021T, Full Marks: 100 (78 Classes), Total Credit:6=5+1(Th+Tutorial),

MODULE –I(Algebra-2) (39 classes)

Course objective*: Learning and application of : 1.concept of matrix and determinant over real or com0plex numbers of arbitrary finite order,in particular, symmetric and skew-symmetric matrices, Orthogonal, Unitary &Hermitian matrices, 2. evaluation of determinant (Laplace's method), 3. consistency and solution of system of linear equations,4. definition,examples,elementary properties of group and subgroup, 4. concept of external binary operation, vector space, linear dependence and independence of finite set of vectors, basis and dimension, 5. Cayley-Hamilton Theorem,6. process of reduction of real quadratic form to its normal form.*

Matrices & Determinants[18]: Algebra of matrices. Transpose of a matrix—its properties. Trace of a square matrix –its basic results. Symmetric and skew-symmetric matrices, Orthogonal, Unitary &Hermitian matrices.properties, cofactors and minors. Product of two determinants(1) . Adjoint, Symmetric and skew-symmetric determinants & its properties(2). Laplace's expansion method for fourth order determinants(1).Inverse of a non-singular square matrix by method of adjoint & method of partition(4) .Rank of a matrix: determination of rank either by considering minors or by sweep-out process. Consistency and solution of a system of linear equations with not more than three variables by matrix method, Cramer's rule.[4]

Introduction to Group Theory[7]—definition and examples taken from number system ,roots of unity, 2x2 real matrices, non-singular real matrices of fixed order(3). Elementary properties of groups (1). Definition & examples of a subgroup—statement of necessary and sufficient condition of being a subgroup— its applications(2). Elementary ideas of Ring & Field as prerequisite to Vector space [2]

Vector Space[7] :Concept of vector space over a field—examples(2), concepts of linear span, linear dependence and independence of a finite set of vectors, subspace(2), Idea of basis of a finite dimensional vector space. Problems on formation of a basis of a vector space (no proof required)(3)

Theory of Eigenvalues[7]: Characteristic equation of a square matrix of order not more than three—determination of eigen values and eigenvectors—problems only(3). Algebraic & Geometric multiplicity of eigenvalues(1). Statement and illustration of Cayley-Hamilton Theorem(1).Real Quadratic forms(2).

Books Recommended:

(1) Linear Algebra--a Geometric Approach -- S. Kumaresan

(2) Linear Algebra- Freidberg, Insel, Spence

(3) Linear Algebra—Rao, Bhimasankaram

(4) Linear Algebra, Concepts and Applications— P.K.Nayak

(5) Linear Algebra, an Introductory Approach— C. W. Curtis

(6) Higher Algebra—Barnard and Child.

(7) Higher Algebra (Classical, Abstract& Linear) — S. K. Mapa

(8) Modern Algebra—Surjeet Singh and QaziZameruddin

(9) First Course in Abstract Algebra— J. B. Fraleigh

(10) Abstract Algebra—D.S. Dummit and R. M. Foote

MODULE –II (Calculus-2) (39 classes)

Course objective:*Learning and application of : 1. Rolle's and MVTs (Lagrange and Cauchy) and their applications,2. Taylor's and Maclaurin's finite and infinite series expansion of elementary functions and their use in determining optimum value of real valued function of real variable, 3. Limit, continuity, partial derivative and differentiability of function of two real independent variables, 4. Euler's theorem on homogeneous differentiable functions, 5.Process of finding optimum of function of not more than three independent variables, 6. Constrained optimization,6. Formation of ODE, 7. First order first degree ode: exact and IF for non-exact, linear and Bernoulli 7. First order higher degree ode and singular solution, 8. Second order linear ode: method of variation of parameters*.

Mean Value Theorems [10]: Statement of Rolle's Theorem - its geometrical interpretation and direct applications .Mean Value Theorems of Lagrange and Cauchy(proof included) and applications(3) . Indeterminate Forms: L' Hospital's Rule: statement and problems only (2).Statement of Taylor's and Maclaurin's Theorem with Lagrange's & Cauchy's form of remainders. Taylor's and Maclaurin's infinite series for functions like exp(x),sin(x),cos(x),(1+ x)n ,ln(1+ x) (with restrictions wherever necessary)(3). Application of the principle of maximum and minimum for a function of a single variable in geometrical, physical and other problems(2).

Functions of two and three variables[10]: Geometrical representations. Limit and continuity (definitions) for functions of two variables(2). Partial derivative: knowledge and use of chain rule. Exact differentials (emphasis on problem solving only)(2). Functions of two variables—successive partial derivatives: statement of Schwarz's theorem on commutativity of mixed derivatives(1). Euler's Theorem on homogeneous function of two and three variables (2). Maxima and minima of functions of not more than three variables—Lagrange's method of undetermined multiplier—problems only(using theory of eigen values)(3).

Working knowledge of Double Integrals [3]. -Problems only

Ordinary Differential Equation (O.D.E.)[16] Formation of ode -exemplification from various fields(2)

First order ode : Exact differential equations, Non-exact differential equations & Integrating factors(no

proof)(4).Linear ode and Bernoulli's equation(2) ,Clairaut's equation: general& singular solutions(2).

Books Recommended:

(1) Introduction to Real Analysis—Bartle, Sherbert

(2) Calculus (Vol. I)—T.M.Apostol

(3) Undergraduate Analysis—S. Lang

(4) Mathematical Analysis— S. C. Malik and Arora

(5) Advanced Calculus(An Introduction to Classical Analysis) – Louis Brand (Dover)

(6) A First Course in Real Analysis—S. K. Berberian

(7) Advanced Calculus—D. Widder

(8) Mathematical Analysis—Elias Zakon

(9) Differential Equations—S. L.Ross

(10) Differential Equations—G.F. Simmons & S. Krantz

(11) Introduction to Differential Equations—Boyce &Diprima.

(12) Mathematical Methods—Potter & Goldberg