**Course: Discipline Specific Elective [Semester-5]**

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| Semester | 5 |
| Paper Number/ Code |  Paper number: 1 Paper code:HMTDS5011T |
| Paper Title | Linear Programming and Game Theory |
| No. of Credits | 6 |
| Theory/ Composite | Theory |
| No of periods assigned  | Th:6 |
| Name of Faculty Member(s) | Prof. Tarun Kumar Bandyopadhyay |
| Course Description/ Objective | * Explains the role of LPP in modeling diverse types of problems in [planning](https://en.wikipedia.org/wiki/Automated_planning_and_scheduling), [routing](https://en.wikipedia.org/wiki/Routing), [scheduling](https://en.wikipedia.org/wiki/Scheduling_%28production_processes%29), [assignment](https://en.wikipedia.org/wiki/Assignment_problem), and design.
* Discusses basic assumptions of LPP with emphasis on linearity of constraints and objective function and discusses ,through examples, the non-linear case.
* Discusses how historically ideas from linear programming have inspired many of the central concepts of optimization theory, such as *duality,* *decomposition,* *convexity* and its generalizations.
* Discusses economical and intelligent Simplex Method to solve LPP
* Explains how LPPs always occur in maximization-minimization dual pairs and its application to Transportation Problem , Assignment problem and Game Theory
* Introduces Game theory as the study of mathematical models of strategic interaction in between rational decision-makers and discusses its applications in different fields of social sciences
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| Syllabus | Introduction to n dimensional Euclidean space En. Directional Derivative and direction of optimization of a linear function f:$E^{2}$ $\rightarrow E$**(3)** General form of Linear Programming Problems: standard and canonical forms. Assumptions behind Linear Programming Problems**(3)**. Graphical Solution of L.P.P and moving hyperplane method: Examples of Finite Optimal Solution; Alternative Optimal Solution; Unbounded Solution; Non-Existence of feasible solution **(3).**Linearly dependent and independent set of vectors in $E^{n}$, Spanning set, Basis, Dimension, Replacement Theorem[Proof Included]**(3)**.Elementary row and column operations on a matrix, Row rank and Column rank of a matrix and the relationship between them [statement only]. Full rank of a matrix, Rank of product of two matrices [statement only], Determination of rank of a matrix using elementary row/ column operations, Invertibility of matrices through rank **(3)**.Solution of system of linear equations: Basis matrix and Basic solution: examples. Feasible solution. Degenerate solution. Reduction of a feasible solution to a basic feasible solution with proof **(3)**.Convex Combination of finite number of points in $E^{n}.$ Convex sets and their properties [Proof included]. Examples **(2)**Extreme points and Boundary points of a Convex set: Examples. Every extreme point of a convex set is a boundary point of the set though not conversely **(3).**Hyperplanes. Halfspaces : Closed and Open. Hyperplanes and Halfspaces are convex sets [Proof Included]. Convex set of feasible solutions of a system of linear equations and linear inequations **(2)**Directions and Extreme directions of a Convex Set. Extreme Ray. Finding Extreme directions of the convex set X= {**x**: **Ax**$\leq $**b**, **x**$\geq 0$}, **x**$\in E^{n}.$ **(2)**Polyhedral Sets and Polyhedral Cones. A point **x**1 of X= {**x**: **Ax**$\leq $**b**, **x**$\geq 0$} is an extreme point of X iff **x1** lies on some n linearly independent defining hyperplanes of X [Proof included] **(3)**Caratheodory’s Representation Theorem of the Polyhedral Set X**=** {**x**: **Ax**$\leq $**b**, **x**$\geq 0$} in terms of extreme points and extreme directions(statement only)**(3)**Correspondence between b.f.s of a system of linear equations and extreme point of the corresponding convex set of feasible region[Proof Included] :examples**(2)**.Simplex Method: Its Algebraic and Geometric Aspect **(2)** Degeneracy and Cycling. Criteria for improvement and optimality of objective function; Criterion for unbounded solution. Computational Aspect of Simplex method: Simplex table. Examples **(4)**Obtaining initial b.f.s. Artificial variable. Charne’s Big M Method. Two-Phase Method **(4)** Duality Theory. Standard form of primal and dual l.p.p. dual of the dual LPP is the primal LPP (proof included). Weak duality Theorem, Fundamental Theorem on Duality(proof included) and their applications. Simultaneous solution of primal and dual l.p.p.s:examples**(4)**. Complementary Slackness Theorem (proof included) **(1)**Transportation Probem: Mathematical formulation. A balanced TP has an optimal b.f.s. In a balanced TP having m origins and n destinations (m,n$\geq $2), number of basic variables in a b.f.s. is at most m+n-1**(2)**. Loops in Transportation Table and their properties : a subset of the columns of the coefficient matrix of a TP are linearly dependent iff the corresponding cells contain a loop(with proof)**(3)**. Obtaining initial b.f.s.: North-west corner rule and Vogel’s approximation method **(3)**. Derivation of criteria of entering and departing vector and test of optimality of a balanced transportation problem **(2)**. Unbalanced transportation problem. Resolution of degeneracy**(2)**The Assignment Problem: Mathematical Formulation. Dual Assignment Problem. The Reduced Matrix. Modifying the Reduced Cost Matrix**(2).**Use of Complementary Slackness Theorem in obtaining optimal solution to an Assignment Problem. Computational Procedure**(2)**. Unbalanced assignment problem. Conversion of Maximization Assignment Problem **(2)**.Two Person zero sum game. The Saddle point and the maximin-minimax principle. Relation between maximin and minimax values**(2)**.Games without saddle point: Mixed strategy**(2)**.Graphical Method of solving nx2 and 2xn games **(2)**. Dominence property: generalised dominance**(1)**. Reduction of a game problem to a LPP. Fundamental Theorem of Rectangular Games **(2)**. Examples **(1)**. |
| Texts  | Linear Programming: P.M.Karak |
| Reading/Reference Lists | 1. Linear Programming and Network Flows: Bajara & Jarvis
2. Linear Programming: D.J. Bhattacharya
3. Linear Programming: G.Hadley
4. Operations Research : An Introduction: Hamdy.M. Taha
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| Evaluation  | CIA: 20End-Sem: 80[ 40 +40] |