**Course: Discipline Specific Core [Semester-6]**

| | |
|---|---|
| Semester | **6** |
| Paper Number | **Paper no:13[HMTCR6131T]** |
| Paper Title | **Differential Equations-III and Metric Spaces-II** |
| No. of Credits | **6** |
| Theory/ Composite | **Theory** |
| No of periods assigned | **Th: 6** |
| Name of Faculty Member(s) | **Prof. Diptiman Saha**<br><br>**Prof. Tarun Kumar Bandyopadhyay** |
| Course Description/ Objective | • To learn the existence and uniqueness of solutions of ODE.<br>• To study the classifications of second order PDE and their simple methods of solutions.<br>• To study some simple Cauchy problems and their geometric insights.<br>• Learning and application of concept of Continuity<br>• Concept of compactness with reference to metric space and it is preserved under homeomorphisms.<br>• Concept of connectedness with reference to metric space and it is preserved under homeomorphisms. |
| Syllabus | **Differential Equations-III (39 Classes)**<br><br>Existence and uniqueness of solutions of ODE, Picard's theorem, Maximal interval of existence and existence global solutions (emphasis should be given on problems) (10).<br><br>Derivation of Heat equation, Wave equation and Laplace equation. Classification of second order linear partial differential equations as hyperbolic, parabolic or elliptic. Reduction of second order Linear Equations to canonical forms. [9]<br><br>Cauchy problem and its geometric insight Cauchy-Kowaleewskaya theorem, Cauchy problem of an infinite string. Initial and Boundary Value Problems, Semi-Infinite String with a |

fixed end, Semi-Infinite String with a Free end, Equations with non-homogeneous boundary conditions, Non- Homogeneous Wave Equation. Method of separation of variables, solving the Vibrating String Problem, Solving the Heat Conduction problem. [20]

**Metric Spaces-II ( 39 classes)**

Concept of continuity of functions between metric spaces. Continuity of real valued functions. Continuity at a point and continuity on a set. Sequential criteria of continuity. Continuity via open sets. Composition of continuous functions is again continuous, algebra of continuous functions. Continuity of vector valued functions (**4**).

Homeomorphic and isometric spaces—homeomorphism is topological equivalence isometricism is metrical equivalence. Topological properties, completeness is not a topological property. Homeomorphic subsets of $R, R^2, R^3$. (**3**).

The concept of Connectedness in a metric space, connected sub sets of real line (**2**), some special connected subsets of plane and space (**2**): Intermediate Value Theorem and its applications. Continuous image of a connected subset of a metric space is connected, connectedness is a topological property. Intersection and union of connected space may not be connected, subspace of connected space may not be connected (**2**). Classification of subsets of reals, plane and space via connectedness (**2**) Product of two connected Space is connected (1).

Bounded sequence may not have a convergent subsequence in a metric space, Concept of sequential compactness (**1**). Bounded infinite subset in a metric space may not have limit points, the concept of BW-Compactness.Equivalence of sequential compactness and BW-compactness (**2**). Compactness via open cover, Sub space of a compact space may not be compact, Intersection and union of compact spaces. Closed subset in a compact metric space is compact (**3**). Finite Intersection Property and its use to prove the non-compactness of Euclidean spaces (**2**) Total boundedness, compactness vs completeness, Banach Fixed point theorem (2). Proofs of four equivalent statements, viz , (a) (M,ρ) is compact MS (b) (M,ρ) is sequentially compact MS (c) (M,ρ)is complete MS and totally bounded (d) (M,ρ) has BWP (**2**).

Compact subsets of the set of reals, plane and space. Compact subsets are closed and bounded in a metric space converse may not be true in general, Heine-Borel theorem (**2**) Closed and bounded sets i.e. compact sets in R must have maximum/minimum elements (**1**).

Continuous image of a compact set is compact. Compactness is a topological property. Real valued continuous function defined on compact domain is bounded and attains its

| | |
|---|---|
| | maxima and minima (**2**). The concept of Uniform continuity of functions, examples, non-examples. Uniform continuous function takes Cauchy sequence into a Cauchy sequence, uniform continuous function has unique continuous extension on the closure of its domain. The distance function from a fixed set in a metric space is uniformly continuous, disjoint closed subsets have positive distance if one of them is compact. Continuous functions defined on compact domain is uniformly continuous (**4**). An open continuous bijection on a compact domain is homeomorphism. Classification of subsets of the set of $R, R^2, R^3$via compactness (**2**). |
| Texts | 1. S. L.Ross, Differential Equations, 3rd Ed., John Wiley and Sons, India, 2004**.**<br>2. Metric Spaces—M. N. Mukherjee |
| Reading/Reference Lists | 1. Tyn Myint-U and Lokenath Debnath, Linear Partial Differential Equations for Scientists and Engineers, 4th edition, Springer, Indian reprint, 2006.<br>2. Martha L Abell, James P Braselton, Differential Equations with MATHEMATICA, 3rd Ed., Elsevier Academic Press, 2004.<br>3. T. Amarnath --An elementary course in Partial Differential Equations, 2nd Ed.,Narosa.<br>4. Walter Kelley & Allan Peterson, Difference equations: An Introduction with Applications , (Academic press,1991)<br>5. R. Kent Nagle, Edward B. Saff, Arthur David Snider --Fundamentals of Differential Equations (8 th Ed.),Pearson<br>6. Topology of Metric Spaces — S. Kumaresan<br>7. Metric Spaces— P.K. Jain & Khalil Ahmed<br>8. Metric Spaces— Satish Shirali & H. L. Vasudeva<br>9. Elements of Abstract Analysis—Micheal O Searcoid<br>10. Metric Spaces— Victor Bryant.<br>11. Introductory Real analysis— M. E. Munroe.<br>12. Introduction to Topology and Modern Analysis— G. F. Simmons |
| Evaluation | **CIA: 20** |

| | |
|---|---|
| | **End-Sem: 80 [40 +40]** |