Course: Discipline Specific Core [Semester-6]

Semester	**6**
Paper Number	**Paper no:14[HMTCR6141T]**
Paper Title	**Series and Sequence of Functions and Complex Analysis**
No. of Credits	**6**
Theory/ Composite	**Theory**
No of periods assigned	**Th: 6**
Name of Faculty Member(s)	**Prof. Tarun Kumar Bandyopadhyay**
Course Description/ Objective	• Explains how concept of uniform convergence helps formation of many theorems of functional analysis, such as the Weierstrass approximation theorem and some results of Fourier analysis; and how it we can be used to construct a nowhere-differentiable continuous function. • Discusses how passage to limit under uniform convergence preserves desirable properties like continuity, integrability and (with additional hypotheses) differentiability of constituent functions and how it allows term-by-term integration and differentiation of a series of functions which has several use in application • Explains how concept of uniform convergence allows defining well-known functions in terms of power series and how their salient properties can be derived using term-by-term integration and differentiation. • This course introduces Complex Analysis as one of the classical branches in mathematics which has useful application in many branches of mathematics, including algebraic geometry, number theory and analytic combinatorics. • It introduces Complex Analysis (as compared to real analysis) having a wealth of beautiful and surprising results which are often strikingly different from results about analogous concepts for functions of a real variable, e.g. if $f:G\rightarrow C$ (G open) is differentiable at all points of G, then f is not only continuous as in the real case, but f is automatically differentiable infinitely often

	in G. • Counter to our intuition (and not having a counterpart in Real Analysis), the course discusses Cauchy's integral formula which surprisingly expresses the value of a holomorphic function at an interior point of a disc in terms of the values on the circumference: Knowing just a tiny bit of a holomorphic function is enough to determine it in a much bigger set. • It studies Holomorphic functions exhibiting some remarkable features. For instance, Picard's theorem asserts that the range of an entire function can only take three possible forms: C, C-{z_0}, or {z_0} for some complex z_0.
Syllabus	**Series and Sequence of Functions (39 Classes)** **Sequence of functions (defined on a subset of R):** Pointwise and uniform convergence. Cauchy criterion of uniform convergence**(4)**. Dini's theorem on uniform convergence**(1)**. Weirstrass' M-test **(2)**. Boundedness. Repeated limits, Continuity. Integrability and Differentiability of the limit function of a sequence of functions in case of uniform convergence**(5)** **[12]** **Series of functions defined on a set:** Pointwise and uniform convergence. Cauchy criterion of uniform convergence**(2)**. Dini's theorem on uniform convergence. Tests of uniform convergence – Weierstrass' M-test**(3)**. Statement of Abel's and Dirichlet's test and their applications**(2)**. Passage to the limit term by term. Sum function: boundedness, continuity, integrability, and differentiability of a series of functions in case of uniform convergence . **(5)[12]** **Power Series (P.S.):** Fundamental theorem of Power series. Cauchy Hadamard theorem. Determination of radius of convergence**(3)**.Uniform and absolute convergence of P.S. Properties of sum function. Abel's limit theorems**(1)**. Uniqueness of power series having same sum function. Exponential, logarithmic and trigonometric functions defined by Power Series and deduction of their salient properties**(2).** Weierstrass approximation theorem **(2)** **[8]** **Fourier Series:** Fourier inversions and Plancherel Theorem . Riemann Lebesgue Lemma. Fourier series of differentiable functions and C^1 functions. Fourier series of functions of bounded variations (statement only) **[7]**

	Complex Analysis (39 Classes) Field structure of complex numbers, field of complex numbers can not be totally ordered, Geometric Interpretation of complex numbers , Topology of the complex plane. Stereographic Projection **(5)**. Recapitulation of sequence and series of real numbers, notion of convergence of sequence of complex numbers: Notion of Convergence. $\{z_n\}_n$ is convergent iff$\{ R_e z_n \}_n$ and $\{I_m z_n\}_n$ both are convergent. Cauchy Condition for convergence. Subsequence in C: every bounded sequence has a convergent subsequence. Analogous results for convergence of $\sum z_n$.Absolute convergence. Statement of (i) Ratio test, (ii) Root test (iii) Dirichlet's test (iv) Abel's test. State only above series and sequence result and ask students to prove as they already have proved in sequence and series of real numbers **(5)**. Function of a complex variable – Exponential, Logarithmic, Direct and Inverse Circular and Hyperbolic functions. Injective and Surjective functions , Concepts of limit and continuity, sequential continuity is equivalent to continuity. Continuity and Connectedness. Continuous function on a compact set is uniformly continuous **(4)**. Analytic Function: Differentiability – definition, derivability implies continuity, differentiability of sum, difference, product, quotients and composition of differentiable functions, Cauchy – Riemann equations are necessary but not sufficient conditions for differentiability of a function at a point in its domain of definition, sufficient conditions for differentiability. Definition of analytic and entire function. Composition of analytic function, Mobius transformations and their elementary properties**(10)**. Harmonic function and Harmonic conjugate : basic results regarding existence and determination of harmonic conjugate. The real and imaginary parts of an analytic function defined on an open subset O of the complex plane are harmonic on O. Milne – Thomson method **(2)**. Sequence of functions and series of functions in C: Pointwise and uniform convergence. Cauchy Criterion. M-Test. Statement of results regarding continuity, analyticity of limit function/sum function in case of uniformly convergent sequence/series of function in C. Power Series as an analytic function: radius of convergence of a power series (Cauchy-Hadamard Form and Ratio Form). Absolute and uniform convergence of a power series strictly within the circle of convergence. A power series and its derived power series have same radius of convergence. Omit the proof of above series and sequence of functions result as those are already covered in the sequence and series of real functions. A power series is an analytic function strictly within its circule of convergence and conversely if f is analytic in a domain D, then f can be represented

	by a power series locally about each point z_0 in D. (statement only) (3). Integration of complex functions along a curve, contour integrations, Cauchy's integral theorem and its applications (10)
Texts	1. S. Goldberg, Calculus and mathematical analysis. 2. Complex Analysis: Ponnusamy.
Reading/Reference Lists	1. K.A. Ross, Elementary Analysis, The Theory of Calculus, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004. 2. R.G. Bartle D.R. Sherbert, Introduction to Real Analysis, 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002. 3. Charles G. Denlinger, Elements of Real Analysis, Jones & Bartlett (Student Edition), 2011. 4. Santi Narayan, Integral calculus. 5. T. Apostol, Calculus I, II. 6. Fourier Series: E.M. Stein, R. Sakarchi. 7. Complex Analysis: Ahlfors 8. Complex analysis of one variable: J.B.Conway. 9. Complex Analysis: E.M.Stein, R. Sakarchi.
Evaluation	**CIA: 20** **End-Sem: 80 [40 +40]**