
St. Xavier's College (Autonomous), Kolkata
Department of Statistics
PRAKARSHO
Volume XII
2020

Faculty Members

One Day National Seminar on
Data Science – The Key to Future
22nd Jul, 2019

National Seminar on Applications of Statistics in Natural Sciences
16th -17th Dec, 2019

Sports Day
1st Feb, 2020

ST. XAVIER'S COLLEGE (AUTONOMOUS), KOLKATA

DEPARTMENT OF STATISTICS

PRESENTS

PRAKARSHO

2020

12TH EDITION

PRAKARSHO

2020

12TH EDITION

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MESSAGE FROM THE PRINCIPAL

Rev. Dr. Dominic Savio, SJ
Principal
St. Xavier's College (Autonomous), Kolkata

"

I am happy to note that the Department of Statistics of our college is successfully publishing its annual departmental magazine, the 2020 edition of Prakarsho.

It is good to see the Department uphold its tradition of excellence and distinction, which the department magazine bears testimony to. Since its inception in 1996 the department has strived to hone new skills and abilities. One of its striking endeavors is that of exploring the prospect of research on the subject into which we get a tiny peek from the articles by students in the magazine.

My hearty congratulations to the entire department, to its faculty and students. I wish them success on their efforts in publishing this issue of the magazine and many more subsequent ones in the years to come.

God bless you all! Nihil Ultra!"

REV. DR. DOMINIC SAVIO, SJ
PRINCIPAL

MESSAGE FROM THE VICE PRINCIPAL

Prof. Bertram da Silva
Vice Principal
St. Xavier's College (Autonomous), Kolkata

"

Once again it is time to congratulate the faculty and students of the Department of Statistics for publishing the latest edition of the departmental magazine, Prakarsho 2020.

The magazine is tangible proof of the department's pursuit of academic excellence and creativity. The magazine is an incentive for research and analysis, and by providing a space for publishing work, it helps train students in the rigours and protocols of academic writing. This has the proper merit of preparing them for the challenges and opportunities of post-graduate study.

If the Department of Statistics has an exceptional reputation and record of academic excellence, it is due in no short measure to its commitment to academic excellence. Prakarsho 2020 is evidence of that commitment."

PROF. BERTRAM DA SILVA
VICE PRINCIPAL

MESSAGE FROM THE DEAN OF SCIENCE

Dr. Tapati Dutta
Dean of Science
St. Xavier's College (Autonomous), Kolkata

"

It is a pleasure to know that the Department of Statistics is ready to bring out the 2020 edition of its annual departmental magazine Prakarsho.

The magazine and the articles within speak volumes about the enthusiasm, motivation, and ambition of the students regarding their field. It is refreshing to see students going beyond their curriculum to make this happen.

I would like to congratulate the department on the success of their current endeavour and I wish them success for their future."

DR. TAPATI DUTTA
DEAN OF SCIENCE

MESSAGE FROM THE HEAD OF THE DEPARTMENT

Dr. Durba Bhattacharya
Head, Department of Statistics
St. Xavier's College (Autonomous), Kolkata

"

It instils in me a sense of immense pride and pleasure to see our students bring out the 12th edition of our Departmental Magazine Prakarsho. Their untiring efforts and relentless energy is indeed praiseworthy.

I would like to extend my heartfelt gratitude to Father Principal, Vice-Principal, Dean of Science and Dean of Arts for their perennial guidance and encouragement. I wish to thank the Program and Publication Committee for their continuous support.

My sincere thanks and appreciation goes to my colleagues, whose efforts and endeavours as a team has helped us come together to unveil yet another reflection of our department."

Durba Bhattacharya

DR. DURBA BHATTACHARYA
HEAD, DEPARTMENT OF STATISTICS

MESSAGE FROM THE STUDENT EDITOR'S DESK

"

Before you turn over the pages, and delve into a carnival of statistical thoughts and ideas, let us first thank the team, whose untiring effort and enthusiasm, has led to the publication of the magazine. Prakarsho is nothing less than a dedicated teamwork, a delightful journey with the flair for excellence, creativity, imagination and zeal that sets a benchmark for itself.

Prakarsho is not just a mere collection of articles; it's that thread that binds the Department Of Statistics. Starting from the professors, to the various working committees namely Editorial, Publication, Designing, Finance, and Cultural, and each & every student of the department- there is hardly anyone who did not lend a hand in this journey.

We would also like to express our heartfelt gratitude to all the authors, sponsors, publishing house and well-wishers for their zealous support and co-operation.

Hence, with immense hope and pleasure, we present to you the 12th Volume of Prakarsho."

Editorial Team

Events Team

Student Covenor
Oiendrila Basak

Student Co-Convenor
Srijan Sen

Events Head
Archik Guha

Cultural Head
Rajdeep Saha

Working Committee Members

Sutirna Chakraborty
Srijit Mondal
Supratim Pal
Soham Biswas
Amrita Bhattacharjee
Rajnandini Kar

Finance Board

Srijit Mondal
Adrija Bhattacherjee
Rajnandini Kar
Mehuli Bhandari
Srija Mukhopadhyay
Tithi Sharon Sarkar

Departmental Report 2020

Student's Achievements

February 2019 - February 2020

6 students from the Department got selected at Indian Statistical Institute, for pursuing M.Stat program and 11 students got selected at different IITs all over India after qualifying the Joint Entrance Test for MSc. in Statistics.

Awards and Recognitions (3rd Year)

1. Vanshika Tantia
   a) Second place in women's double scull for 2000m and 500m in the All India Inter University Rowing Nationals held from 24th February till 1st March 2019 in Chandigarh.
   b) Gold in women's coxed fours in BRC Inter-college Regatta. 2.

2. Debarshi Chakraborty
   a) Has won Bronze(team event)---Amephoria, Amity University
   b) Gold (singles)---Xavrang, SXUK
   c) Champion in XPL 2019 (Table Tennis).
   d) Runner Up in Xavotsav 2020 (Table Tennis).
   e) Represented North Kolkata district in Bengal State Table Tennis Championship 2019 - won Bronze medal in team event.
   f) Represented SXC Kolkata in CU Inter College Table Tennis tournament 2019- won silver medal in team event.

3. Sreeja Deb Ray
   a) 1st prize in Badminton Doubles in annual sports event KHEL organized by NCC.

4. Shibashish Mukherjee
    a) All India Karate competition, 2nd place held on July 12 2019 in New Delhi.

5. Srijit Mondal
    a) Ne8x Literature Fest- Author of the Year 2020.
    b) GFEL- World's top 100 Education Innovators Nominee.

Awards and Recognitions (2nd Year)

1. Ishita Pandey
    a) 1st Place in Discus Throw and Triple Jump.
    b) Awarded Best Athlete in Annual Sports 2020.

2. Soham Majumder and Tishyo Chakraborty secured 1st place in INFINITE DIVERGENCE in SIGMA 2019 organized by St. Xavier's College Science Association.

3. Samiran Ghosh, Sovon Gayen and Purnaloke Sengupta secured 2nd place in BATTLE ROYALE in SIGMA 2019 organized by St. Xavier's College Science Association.

4. Somjit Roy
    a) Runner-up in Inter Collegiate Cricket Tournament, Calcutta University.

5. Supratim Pal
    a) Silver in 4th Inter College Rowing Championship 2019.
    b) Represented INDIA in ARAE Colombo 2014 and received GOLD.

6. Sabuj Ganguly secured 1st place in Creative Writing in SRIJAN 2019.

7. Srijan Sen won the 'Author of the Week' title in Inter College Creative Writing Competition — Bengali Poetry Category by Storry Mirror.

8. Srijan Sen, Soham Majumder, Supratim Pal and Ritoban Sen presented a paper on 'A Simulation Study Of Sample Size Determination For Achieving Normality Of Binomial Distribution' in Indian Science Congress 2020.

9. Srijan Sen, Soham Majumder, Supratim Pal and Ritoban Sen presented a paper on 'A Simulation Study Of Sample Size Determination For Achieving Normality Of Binomial Distribution' in Indian Science Congress 2020.

10. Soham Biswas, Ishani Karmakar, Srijan Sen and Brishti Sarkar presented a paper on 'A Simulation Study Of Sample Size Determination For Achieving Normality Of Beta Distribution' in Indian Science Congress 2020.

11. Soham Biswas, Tishyo Chakraborty, Somjit Roy and Arpita Saha presented a paper on Situations of Supply in the Classical Newsboy Problem in Indian Science Congress 2020.

12. Ritoban Sen, Supratim Paul, Arpita Saha and Brishti Sarkar presented a paper on 'Newsboy Problem: Estimation of Optimal Order Quantity' in Indian Science Congress 2020.

13. **Soham Majumder, Tishyo Chakraborty, Subharanjan Mondal** and **Aditya Paul Chaudhuri** presented a paper on 'Reduction of Stress Induced Diabetes in Urban Population Through Yoga - A Statistical Approach' in Indian Science Congress 2020.

14. **Subharanjan Mondal, Aditya Pal Chaudhuri, Somjit Roy** and **Ishani Karmakar** presented a paper on 'Newsvendor Problem: Alternative Optimality Criteria' in Indian Science Congress 2020.

15. **Pallavi Chakravarty, Ramyani Dutta** and **Souhardya Mitra** presented a paper on 'Statistical Analysis of Antibiotic Effectivity & Sensitivity of Urinary Tract Infruction Causing Bacteria in Urban Population' in Indian Science Congress 2020.

Awards and Recognitions (1st Year)

1. Shantanu Nayek
   a) Gold medal in 'Inter Mission Football League'.

2. Saptarshi Chowdhury
   a) 'Special Mention' award in Intra DPS MUN in UNHRC and Third place in 'Regional Mathematics Olympiad' in Assam.

3. Soham Ganguly
   a) Winner of Inter Departmental table tennis tournament in SXC in 2019.
   b) Member of the College Table Tennis team that won Silver at the Calcutta University Inter College Table Tennis tournament.
   c) Runner's Up- XPL (table tennis).
   d) Champion (Table Tennis)- Bhawanipur Education Society College Fest (Umang).
   e) Champion (Table Tennis)- JDBI Fest (Invictus).
   f) Participated and completed 10Km marathon organized by Tata Steel.
   g) Participated and completed 10Km marathon organized by IDBI Federal Life Insurance.
   h) Won Silver at East Endurance Cycling Challenge(80Km) organized by Cycle Network Grow.

4. Adrija Bhattacharya and Debolina Bhattacharya Winner of 'HRPR' event in Enthusia 2019 conducted by EnactusSXC.

5. Gourav Daga
   a) Third place for TCS Quiz on science and technology.

6. Rajnandini Kar
   a) Third place in Inter department table tennis tournament in SXC in 2019
   b) 2nd in Inter Departmental Table Tennis tournament.
   c) 3rd in Inter Department Football tournament.

8. Sandip Chakraborty
    a) CU Inter College Team- Silver.
    b) Umang(Inter College fest at Bhawanipur college)- Gold (team event).
    c) Xavotsav- Bronze (Singles)
    d) Invictus (JD Birla Inter College fest)- Gold (Doubles).

Participation and Certificates (3rd Year)

1. Debarshi Chakraborty
    a) Represented North Kolkata district team in West Bengal Inter district Table Tennis championships.
    b) St. Xavier's College Team throughout the season.

2. Bisakh Banerjee was selected to attend a summer camp in Mathematics in Chennai Mathematical Institute (CMI) in June 2019 organised by National Board of Higher Mathematics.
3. Tanuj Sur co-authored "An Innovative Method to Calculate the Economic Development Index of Important Cities in West Bengal from Satellite Imagery", published in International Journal of Computer Sciences and Engineering.

4. Ankita Prakash
    a) One of the 120 selections in India for the Explore ML program of Google India.

5. Niladri Kal was selected for JBNSTS Talent Enrichment Program from June 30th to July 7, 2019.

6. Dattatreya Mitter, Arghyamalya Biswas, Annesha Deb and Srijit Mondal co-authored "URO VABNAR JONMOSTUP" a poetry book published by Starlet Publication.

7. Sreejit Roy got a poem published in yearly magazine "MOITREYO MANDAS".

8. Dattatreya Mitter got his poem published in the book "C/O ANTORIK".

Participation and Certificates (1st Year)

1. Rajnandini Kar
    a) Certificate in Economics quiz from SRCC.
    b) Certificate in guitar from 'Trinity College of London'.

2. Rohit Dutta got certificate for 'Tabla' from Pracheen Kala Kendra.

3. Utsyo Chakraborty got certificate for piano from 'Trinity College of London'.

4. Tanishi Parasramka Qualified and got a certificate for the top 5 in Inception, the fest by XCS.

Placement Details

This year, a lot of students from the final year of the department bagged lucrative placement offers, through the SXC Placement Cell.

1. **Archik Guha** and **Ayushi Biyani** were offered the post of Associate Analyst at Deloitte USI.
2. **Nandini Agrawal** was offered the post of Tax Analyst at Ernst & Young Global Delivery Services.
3. **Sayak Giri** was offered the post of Trainee Data Science Executive at Spring & River.
4. **Sutirna Chakraborty** was offered the post of Associate Analyst at Swiss Re.

Departmental Activities

Coverage of Epsilon Delta, 2019

On 20th March, 2019, Department of Statistics, St. Xavier's College (Autonomous), Kolkata, embarked upon an era of success and achievements as it was glorious to behold the third edition of the annual departmental event "Epsilon Delta". The inaugural ceremony being hosted by respected Principal and Rector Rev. Fr. Dr. Dominic Savio, S.J. and other dignitaries of the college administration, the luster of the event reached it's brilliance with the release of the 11th volume of the departmental magazine "Prakarsho", which imbibed the ideas of Prof. Sugata Sen Roy of Department of Statistics, University of Calcutta and Prof. Bimal Kumar Roy , former director of Indian Statistical Institute (ISI), Kolkata. The magazine also showcased the statistical as well as academic approaches of the young minds that is the students of the department itself from 1st, 2nd and 3rd year in the form of articles. The seminar was marked by the great presence of Prof. Bikas Kumar Sinha , ISI Kolkata who introduced and conveyed some well known and important aspects Departmental Activities: of Game Theory. Along with Prof. Sinha, the occasion was enlightened by Prof. Gaurangadeb Chattopadhyay of University of Calcutta. Not only the seminar witnessed great professors and dignitaries but also provided ample opportunities to the students of Statistics Department and other departments of the college as well, as students from other colleges, proved their academic excellence and diversity through events like "Proectura" (Paper Presentation), "Xposure" (Online Photography) and "Inquisitive" (Quiz competition). The daylong seminar earning great appreciation and engraving the memories created within the faculty members and students of Statistics concluded with a cultural programme performed by the students of the department itself.

Data Science Seminar, July 2019

One day National Seminar on " Data Science- The key to Future", was jointly organized by the Departments of Computer Science and Statistics, in collaboration with The Data Science Foundation on 22nd July 2019, at Fr. Depelchin Auditorium. Pro-Vice-Chancellor for Academic Affairs of University of Calcutta, Prof. Asis Kr Chattopadhyay was the Chief Guest and Dean of Faculty Councils for PG studies in Engineering and Technology, University of Calcutta, Prof. Amlan Chakraborty was the Guest of Honour. Mr. Kaustav Majumdar and Mr. Gautam Banerjee, from Data Science foundation were the first two speakers of the day. Prof. Sourabh Bhattacharya from Indian Statistical Institute and Mr. Sanjoy Karmakar from IBM were speakers for the third and fourth session respectively. More than 400 students from our college, along with teachers and professionals from Other Colleges/ Universities attended the seminar.

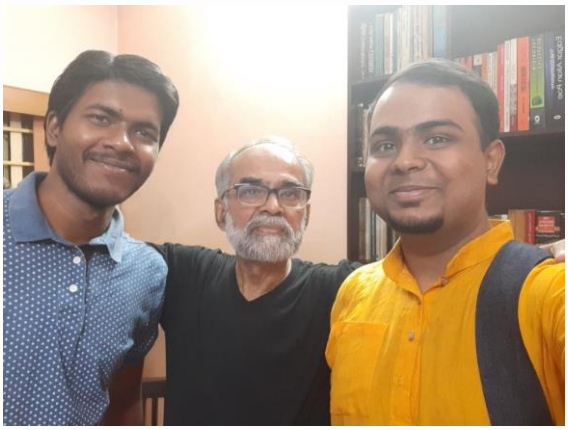
Two Days National Seminar on Applications of Statistics in Natural Sciences, December 2019

Two days National Seminar on "Applications of Statistics in Natural Sciences", was jointly organized by the Departments of Statistics and Physics in collaboration with IUCAA Centre for Astronomy Research and Development (ICARD), Kolkata on December 16th and 17th,2019. Pro-Vice-Chancellor for Academic Affairs of University of Calcutta, Prof. Asis Kr Chattopadhyay was the Chief Guest for the seminar. Oraland poster presentation showcasing the research of College and University teachers and Research Scholars from different fields of Natural Sciences were carried on both the days. In total 15 oral presentations and 12 posters were presented. Best oral presentation award was achieved by Debashish Chatterjee of ISI, Somsubhra Ghosh of IACS, Kolkata and Sreetama Das Choudhary. Best Poster award was won by Avinanda Chakraborty of Prsidency University. In addition, the seminar also had specialized sessions by invited eminent speakers like Prof. Ayanendranath Basu, Prof. Saurabh Ghosh and Prof. Supratik Pal from Indian Statistical Institute and Prof. Rajesh Kumble Naik from IISER Kolkata. More than 100 teachers and research scholars from St. Xavier's College, Kolkata and Colleges/Universities from other states attended the seminar.

In Conversation With Professor Amit Ghosh

Prof. Amit Kr. Ghosh was one of the professors to witness the Department of Statistics, St. Xavier's College, Kolkata grow from its initial years. Hence, the students decided to have a candid conversation with him about Statistics as a discipline and how it shaped it life as a whole, and of course, his experience as a teacher in this College. Hence, this interview was conducted by Dattatreya Mitter and Soumyabrata Bose, on behalf of the editorial committee, on the 20th of February, 2020, at Prof. Amit Ghosh's residence.

(The following is the written and abridged form of the interview , that has been prepared, in consultation with Prof. Amit Ghosh.)



Interview

Dattatreya & Soumyabrata: Sir, first, thank you so much to manage a time for us out of your busy schedule. We have come to have a candid conversation with you regarding Statistics and it's impact on your life, for our departmental magazine Prakarsho.

Prof. Amit Ghosh: Welcome. Thank you for coming and taking interest in having a talk with me. Please convey my sincere thanks and gratitude to all your teachers and friends in the Department for giving me this wonderful opportunity to speak my mind. Best wishes.

Q1. Sir, please share something about your childhood and school days.

I was born & brought up in a large joint family comprising of people of three generations living together in an ancestral house located in the heart of North Calcutta. Our childhood and adolescence was enwrapped in a Hindu Bengali middle class social milieu with all its characteristic ethos, customs, dreams and despair. The rat-race to grab a creamy slice of 'success' at the cost of all other moral and social values did not engulf the mindscape of our parents and teachers. We didn't experience much tension & anxiety about the prospects of our future. Rather we enjoyed enough freedom of thought and many poised moments to stand & stare.

I was a student of Metropolitan Institution(Main) in a close proximity to our residence. Its founder-headmaster was Iswarchandra Vidyasagar and it was a part of the adjacent Vidyasagar College as we find in the case of our SXCS and SXC today. It was a traditional Bengali medium school that was truly a 'neighbourhood' school bonding teachers and generations of students & their parents living around its sacrosanct building. I wonder today how we were blessed with a bunch of dedicated & distinguished teachers who as true disciples of Iswarchandra preached by their deeds the lofty idea of 'plain living & high thinking.' They tried their best to imbue our mind with all kinds of moral values, patriotism, thirst for knowledge and above all an aspiration to serve the country & its people. Till this day I recall them with profound regard &tears of gratitude.

Ours was system of school living examination at the end of class 11 and the syllabi were composed of all those from class 9 to 11.In our Metropolitan school, we had a privilege of getting routine lectures on some portions of our syllabi of Physics, Chemistry, Mathematics & English by the renowned professors of Vidyasagar College, City college & Scottish Church College which were some of the premier colleges in North Calcutta of our time.

Q2. Sir, how were your college days?

After my H.S. Exam I took admission in The Presidency College which was at a stone's through from our house. I spent my days there in the late 60's & early 70's of the last century. That was a great time for me to learn. The whole world was witnessing a stormy time of deep economic crisis, imperialist aggressions, peasant revolts, workers strikes and student upsurges. This turbulent time impacted our life in the college campus. That very ambience helped us to grow as adult citizens responsive to the issues & questions about the age-old traditions, social injustices and people's struggle against economic exploitation and political repression. All these left an indelible imprint on our young minds.

Q3. Sir, why did you choose Statistics as your Honours subject after the completion of HS Examination?

To tell you the truth, in those days people around me had little idea about the exact nature and scope of Statistics. The teachers of our school inculcated in me a keen interest in Mathematics. Also, I and some of my friends were inspired by Prof Prasanta Chandra Mahalanobis' vision of Statistics as a novel mathematical technology of the 20th century for the development of science and society. We took much interest in the articles

on this subject published in 'Jnan O Bijnan', a popular monthly Bengali magazine for school students of our time. However, as I go down the memory lane to relook into the choice of the subject, it now seems to me that the key issue was something else. The unknowns rather than the knowns beaconed the young minds.

Q4. Sir, tell us a few words about your experiences in the Statistics Department of the Presidency College.

In fine, that was great. It had the distinction of introducing Statistics as a Major subject in the undergraduate curriculum in 1944, the first of this kind in India. In our time we were privileged to have one of the star faculties of the college: Prof A Bhattacharya, Prof A M Goon, Prof M K Gupta, Prof B Dasgupta, Prof B Das, Prof D Basu and Prof A S Nag. It was a bunch of most distinguished and dedicated scholars and teachers that one could imagine! Prof A Bhattacharya, as you know, is famous for his enunciation of 'Bhattacharya's Inequalities' in statistical inference that provide more stringent inequalities than that by 'Rao Cramer Inequality'. We witnessed the evolution of the 3rd revised & enlarged edition of 'Fundamentals' and the 1st edition of 'Outlines', two universally acclaimed text books for the undergraduate Statistics as a Major, almost chalked out by the legendary trio Professors Goon
Gupta-Dasgupta on the sliding black boards in our Honours classes. The academic atmosphere in the Department was conducive to a meaningful student-teacher interaction. The average size of the Statistics Honours classes over the years never exceeded 15. The entire Faculty was easily accessible to the students from morning to late evening hours notwithstanding the regular turmoil in the college campus. The diversity of life in the campus made Presidency College a vibrant centre of higher education in our time.

In this connection, it may be mentioned that Prof A M Goon and Prof A S Nag worked for many years as Guest Professors in our St. Xavier's College after their retirements from The Presidency.

Q5. Sir, what kinds of text books did you use to manage your study of the Statistics Honours papers? Do you think the 'core' of it as presently taught is significantly different from that of your time?

In our time no text books were easily available to the students of our country that could do justice to the standard of the CU Statistics Honours syllabus. Most of the available books were either too advanced and/or difficult or too simple or terse to satisfy the real needs of the students. We had to fall back upon 2 volumes of 'Fundamentals' and the rigorous lecture notes delivered by our teachers. These sources were thinly supplemented by books from the college/department library such as:

- 'Statistical Methods' by Mills
- 'Introduction to the theory of Statistcs' by Yule & Kendal

for statistical methods;

- 'Introduction to Mathematical Statistcs' by Hogg & Craig
- 'Advanced theory of Statistics' by Kendal & Stuart

for statistical inference;

- 'Introduction to Mathematical Probability' by Uspensky
- 'Introduction to modern Probability Theoy & its applications' by Feller

for probability theory;

- 'Introduction to linear statistical models' by Graybill
- 'Design & analysis of experiments' by Kempthorne

for ANOVA & DOE;

- Algebra' by Ferrar
- 'Finite Difference' by Freeman
- 'Numerical Analysis' by Scarborough
- 'Mathematical Methods' by Courant

for mathematical methods in the Honours course;

- 'Sample Survey: Theory & Methods' by Murthy

for sampling techniques, etc.

For applied statistics topics and Indian official statistical system we had no standard text books/manuals other than the Volume2 of 'Fundamentals' though Spigelman's 'Introduction to Demography' and 'Applied General Statistics' by Croxton & Cowden were available on the book shelves of the college library. Most of the books I have just mentioned are not in vogue now except Feller's elegant and all-time great texts on probability theory. Also, the book by Hogg & Craig has proved its worth over time as a very useful text book with proper rigour and are still in use.

As to the 2nd part of your question I can say that the 'core' is not radically different today from that of our time though the syllabi have been undergoing a lot of notable changes since 1990's, first in the CU and then in the autonomous SXC.A bulk of the algebra of determinants, calculus of finite differences and numerical analysis was dropped from the syllabus of mathematical methods for Statistics and was replaced by a considerable amount of real analysis and linear algebra. In statistical theory, Pearsonian System of curves and Edgeworth series expansions are no longer taught. Various kinds of abridged life tables in Demography, changing seasonal indices &periodogram analysis in Time Series, Dodge-Romig sampling plans in SQC and some other non-core materials have been removed from the old syllabi.

Q6. Sir, how did you manage the complicated and heavy numerical computations in your Statistics Practical classes without the tools of coding, softwares and computers that we are now used to?

Definitely, you are much privileged in a modern statistical computing environment. Our environment will appear to you as rather antique. We had to use old type-writer like 'FACIT' machines for arithmetical operations; 'Barlow Tables' for roots/powers/reciprocals of numbers; 'Chamber's Seven-figure Log Tables; 'Biometrica Tables' by Pearson Heartley/ 'Fisher-Yates Tables' for parametric statistical inference and 'Owen's Tables' for non-parametric inference. We had to use Squared papers along with Graph-papers for numerical computations, tabulations & diagrammatic representations, and all such jobs were to be done with a pencil. I do not hesitate to say that 90% of our Practical class hours were spent in the management of numerical computations on readymade stale data, leaving little time for us to explore live data and learn therefrom. The only positive quality that such a practice imparted to us was a skill that could make efficient 'algorithms' for elaborate numerical computations. I sincerely believe, your statistical intuitions & skills would thrive in a much better mode in your modern computing environment along with your easy access to Internet resources.

Q7. Sir, how would you like to appreciate today the under-graduate teaching-learning process of your time?

First: that was definitely a teacher-centric process. Lectures by teachers and notes taken down by students passively were the only two components that made the process. No other modes were seriously explored in those days. Second: there was an inherent 'bias' towards 'theory' in one sense. As Statistics is a common research methodology for scientific inquiries cutting across various disciplines, its structural features bear the hallmarks of the said methodology, i.e. quires, plausible hypotheses, relevant data, information measured and inductive verifications, all carried out in an unbroken chain that repeats and expands. This continuum is known in modern science as 'Hypothetico-Deductive-Inductive' method. We had given much emphasis on the deductive manipulations of 'measures' and rigorous mathematical treatments of 'methods' in isolation from actual data and real questions to be resolved, at the cost of 'exploration' of data, 'heuristic' arguments and inductive modelling. I strongly believe that with the advancement of Statistics as a distinct discipline and its computing environment, we had, in this regard, made a lot of progress in the recent past; and I am sure we would be more able, over time, to mend the process and transcend the limitations in the interest of students.

Q8. Sir, please tell us in a few words your experiences in Post Graduate classes?

I obtained my Master's degree from the CU. The Statistics Department was situated on the 5 th floor of a modern multi-storey building at Ballygunge Circular Road. The department was established in 1941 as the first of this kind in India. When we entered this department its faculty had a very rich profile made by some of the most eminent scholars and teachers of that time. To mention a few names: Prof H K Nandi, Prof P k Banerji, Prof B Adhikary, Prof S K Chatterji, Prof S P Mukhopadhyay, Prof A chaudhury, Prof B K Sinha. They elevated the department to the most respectable national level through their research work in the field of statistical theory & methods and regular rigorous teaching in the Post Graduate classes. Prof S P Mukhopadhyay introduced Operations Research in the conventional curriculum of Statistics and taught the same as a special paper offered to us. He did pioneering works in Industrial Quality Management and extended their lessons to enlighten the industrial managements in India and abroad.

As to the availability of text books, the mainstream of teaching-learning process and the routine practices in practical classes, I think today that no new window was really opened up after our under graduate days. The deep ruts of the tradition remained unaltered.

Q9. Sir, please tell us something about the scopes for learning Statistics and securing professional jobs in your time. Do you think that the scenario is different at present?

Definitely, the present scenario is quite different in multiple respects. In our days the number of educational institutions offering UG and/or PG programmes were very limited in number. In our State only 3 colleges under CU, i.e., Presidency, Asutosh, Narendrapur RKM offered UG programmes and ISI, both UG &PG. Outside the State, The University of Pune also offered such programmes. There was

paucity of research fellowships with a reasonable stipend in our country and abroad. The opportunity for doing Master's degree outside the country was also thin. In the academic arena of our country the faculties of science & technology stood far away from modern interdisciplinary research activities. There were no national or multinational corporations with their Analytic wings on our soil seeking professional statisticians for research & development. The Indian Statistical services (ISS) was the major service provider for professional jobs through its public examinations. It recruited statisticians for the Indian Statistical Offices. Even in this field the recruitments were neither frequent nor large in numbers. Also, the pay packet and facilities were not so lucrative. Extensive changes have taken place in all these spheres widening the scopes for higher learning and ushering in new job prospects. Even the Government of India has established a separate Ministry Of Statistics & Programme Implementation (MOSPI) and a Central Statistical Commission (CSC).It has also extended and upgraded the ISS.

It is not difficult to see why and how such changes have taken place. You are living in an age of digitized information and Statistics is the key technology for the most efficient navigation through the oceans of such information. Brace up for a voyage.

Q10. Sir, tell us in brief how do you take on the recent trends and prospects of Statistics.

The 21st century, i.e., your millennium, is witnessing an enormous 'explosion' of information due to an all pervasive digital revolution. The enormity of its impact induces 'paradigm shift' in almost every domain of our science & technology. The conventional frontiers of the distinct domains are intertwining giving rise to redefinitions and new contours. Statistics is no exception. In the interdisciplinary fields of Data Science or Big Data Analytics, diverse disciplines such as Mathematics,
Statistics, Machine Learning & Deep Learning Algorithms are mingled reinforcing one another. It leads to more efficient management & manipulation of data of enormous volume, variety and velocity. In this connection, I may also mention that Bayesian Statistical Inference is reinventing itself extending the horizon of classical inductive inference.

Q11. Sir, how do you appreciate our Choice Based Credit System (CBCS)?

CBCS is definitely a student-centric new curriculum as compared to our old annual system based on rigid patterns and marks. It offers multiple benefits to students according to their needs and inclinations. Of course, when I make this comment, I assume that infrastructure, faculty strength and faculty upskilling are up to the mark. Moreover, its introduction and success call for much attention and vetting in the following three areas

1) Framing and revision of syllabi from the perspective of their 'dynamic core' rather than their 'additive or cumulative' growth.
2) A student-centric and innovative teaching-learning mode.
3) Compatibility of the 'volume' and the 'difficulty level' of the syllabi with a rationally worked out 'average study-hours' (class & self-study) for a student per semester. This is absolutely necessary for proper assimilation of the lessons of each semester syllabi and their retention over semesters.

Q12. Sir, we have heard from our professors and ex-students that you are a great teacher; we would like to know how did you get interested in teaching.

I shall answer your question but with a caveat. I am not a great teacher. I am miles away from any kind of greatness. What one can say, I tried sincerely to deliver my best. That is it. No more and no less. I was impressed and inspired by some great teachers whom I came across in my life and adored much. I tried to imbibe their spirit and in my humble capacity thought to tread on their heels. When I completed my Master's degree, I realized that the immense scope of the subject was not fully explored in our country. We need more research & training institutes, more scholars and teachers to carry the task forward. We need more skilled hands & brains. This vision charted the roadmap. The rest were intimate details of a personal life.

Q13. Sir, you had spent a long time in our St. Xavier's College. How were the days?

In fine, wonderful. I have no hesitation to say that I spent some great moments with my students and colleagues in the Department and the College at large. I shall cherish the memories through the rest of my life.

St. Xavier's always stands for something Big. It upholds plurality and secularism in the Indian culture. Students joined this college and built bonds with one another cutting across the boundaries of regions, languages, religions, castes & creeds. I learnt a great lesson from them. The Department & SXC was my second home. But I knew it was so to the students of our Department also. I still remember the day when some of them entered the department room on the top of the college building and proposed their teachers without hesitation that they had planned to publish an annual departmental academic magazine on a regular basis and they had also thought a name for it: Prakarsho. That was the beginning of a journey.

Q14. Sir, lastly, what are your advices to students like us?

See, I believe, I can't advise you. I can only share my experiences and thoughts with you so that you can judge and reach somewhere. Here some of my stray thoughts for you.

Preparations for a war to win and those for battles to fight through, though interwoven, are distinct. To strike a balance between these two ends is the key to success...
Stay connected with people around you. Make bonds with trust and compassion...
Accept diversity around you. Get ready to accept differences.
Dare to know. Dare to learn. Dare to discover your inner strengths and weaknesses. And in a fast changing world, also dare to unlearn to remake yourself...
Dare to dream. Dare to pursue your dreams with passion. You can give your best to others only when you love what you do. Always remain true to yourself...
Lastly, I would like to share with you a few lines from the great Arabian artist-philosopher Kahlil Gibran:

"Your daily life is your temple and your religion. Whenever you enter it take with you your all."

Dattatreya & Soumyabrata: Thank you, Sir.

A Review On the Applications of Regression Analysis

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Regression analysis is a statistical process to find out the relationship between some factors of interest in various purposes. It helps us to answer many questions like: Which factors have the greatest impact? Which factors can be ignored? How are those factors interrelated with each other? And, the most importantly, how certain are we about those factors at all?

In regression analysis, those factors are referred to as variables. These variables are classified into two types-

1. Dependent variable — the main factor that we are trying to predict, 2.Independent variables — the factors that we suspect to have an impact on our dependent variable.

Now in our discussion, we are not going to discuss much about how to draw a regression line, why it is called the ' Best explanation of the relationship between the independent and dependent variable' and all, but we are going to discuss the real life fields in which Regression Analysis has a vast impact.

Regression Analysis in Business Forecasting:

Predictive Analysis is the most prominent application of regression analysis in business. It is based on forecasting business opportunities and future risks. Demand analysis deals with prediction of the demands of a certain product. Regression analysis is a process that is heavily relied on by the Insurance Companies to estimate the credit standing of policyholders.
Regression Analysis also increases operation efficiency in Business. For example, using Regression Analysis, a factory manager can understand the relation between productivity of certain product and other factors and improve the product's demands in market.

Regression analysis provides a scientific view to various business managements by reducing huge amount of raw data into information that is actually required, that is how regression analysis leads the way to more perfect decisions. This tool is used to test a hypothesis before diving into implementation. Regression analysis can prevent mistakes also.

Regression analysis also may uncover some previously unnoticed patterns while finding a relationship between different variables. For example, it may uncover the fact that the demand of specified product is usually increased in a particular season of the year, that was may not be noticed beforehand.

Regression Analysis in Machine Learning

Regression analysis helps to prepare schemes for given situations well in advance and then analyze its probable outcomes. As machine learning is based on predictions, Regression Analysis helps it in a grand way. The actionable information that comes from Regression Analysis helps companies to make their strategies.

One of the biggest advantage of using Linear regression model is to forecast trends, patterns and making useful decisions in Machine Learning. Moreover, these decisions can be used further in Machine Learning. Its accuracy level is high, efficient enough and fast.

Regression Analysis in Finance

Multiple Regression Analysis is preferable to forecast financial statements for a company. It is usually for determining the changes in certain assumptions of business that will impact in future. For example, the correlation between the number of employees employed by a company, the number of branches they have and the revenue of the business, may be very high.

Regression Analysis in Biological Experiments

There are several examples in biological experimentation where Regression Analysis is used.

In the example below, Regression analysis has been used to understand the relationship between the petal length and petal width of the flowers of Iris versicolor. The output for one such analysis is shown below.

We have set Petal Width to be the independent variable and Petal Length to be the dependent one. Hence, we will try to predict Petal length on the basis of Petal Width. Coefficient of determination, r^2, comes out to be 0.61. This value signifies that knowing the width of a petal should enable us to make a very good estimate of petal length. As we know, F-test compares the variation explained by the regression line to the residual variation, and the p-value from the F-test gives us the probability that the slope of the regression line is zero (i.e., the null hypothesis). The F-value is of 77.93 and p-value is <0.0001. The low p-value indicates that the probability that the two variables are not related is vanishingly small. Therefore, the equation for our line is:

Petal Length
= (Petal Width * 1.8693) + 1.7813

```
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)           1.7813     0.2838   6.276 9.48e-08 ***
versicolor$Petal.Width   1.8693     0.2117   8.828 1.27e-11 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2931 on 48 degrees of freedom
Multiple R-squared:  0.6188,    Adjusted R-squared:  0.6109
F-statistic: 77.93 on 1 and 48 DF,  p-value: 1.272e-11
```

Regression Analysis in Agriculture

In this field, the variables considered for Regression analysis are **Annual Rainfall (AR), Area under Cultivation (AUC), Food Price Index (FPI)**. Here, crop yield is a response variable which depends on all these ecological factors. There are many more factors like **Minimum Support Price (MSP)**, Soil Parameters, Weather Conditions, etc. that can affect the production of various crops and by using techniques like data mining to analyze the factors influencing the yield the research work can be extended.

Regression Analysis in Artificial Neural Engineering

The artificial neural engineering models were built to forecast shelf life of instant coffee drink like regression models. Both the models were compared with each other. The investigation shows that multiple linear regression model was superior over radial basis model for these type of investigations in Artificial Neural Engineering.

Besides all these fields, there are several other fields where Regression Analysis plays a vital role such as in Researches, in Chemical & Pharmaceutical Experimentation, in Economics, in Mechanical Engineering & many more and that is why **Regression Analysis** is necessary in Multi-Disciplinary Fields.

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Finding the Summation of a Famous Series Using Probability Distributions

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It remained an 'Open Problem' to evaluate the summation $\sum_{n=1}^{\infty}\frac{1}{n^2}$, for almost 90 years. This problem is also known as '**Basel Problem**' and Euler was the first person to provide a complete solution of this problem. But people always looked for new, interesting and enlightening approaches to solve this same problem. An elementary but interesting approach is to use the concepts of 'Probability Distributions'.

An important distribution

For our purpose we will define a 'Folded Cauchy Distribution'. A random variable X is said to follow a 'Folded Cauchy Distribution' with location parameter 0 and scale parameter 1 if the PDF of X is given by

$$f_X(x) = \frac{2}{\pi(1+x^2)};\ x > 0$$

Sketch of the proof

Suppose, X_1, X_2 are two independent and identically distributed random variables following a 'Folded Cauchy Distribution' with location parameter 0 and scale parameter 1.Let us define another random variable $Y = \frac{X_1}{X_2}$. We will try to find $P(0 < Y < 1)$ in two different ways, namely, by inspection and by using joint density function. Then we will expand a geometric series and integrate it term by term to establish the result. As we know that $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is equal to $\frac{\pi^2}{6}$, here we will try to develop a proof of this well- known fact by the way we described here.

Proof

The joint distribution of X_1, X_2 is given by,

$$f_{X_1,X_2}(x_1,x_2) = \frac{4}{\pi^2(1+x_1^2)(1+x_2^2)};\ x_1, x_2 > 0$$

Let us consider $y = \frac{x_1}{x_2}$ and $z = x_2$. So, we get $x_1 = yz, x_2 = z$.
The ranges of y and z are $0 < y < \infty$ and $0 < z < \infty$.
The modulus of the Jacobian of the transformation is z . So, the joint PDF of Y and Z can be defined as

$$f_{Y,Z}(y,z) = \frac{4z}{\pi^2(1+y^2z^2)(1+z^2)};\ y, z > 0$$

Hence, the marginal PDF of Y is given by,

$$f_Y(y) = \frac{4}{\pi^2}\int_0^\infty z\frac{1}{(1+y^2z^2)}\frac{1}{1+z^2}dz\ ;\ y > 0$$

$$= \frac{2}{\pi^2(y^2-1)}\left[\ln\left(\frac{1+y^2z^2}{1+z^2}\right)\right]_0^\infty; y > 0$$

$$= \frac{2}{\pi^2(y^2-1)}\ln(y^2)\ ; y > 0$$

$$= \frac{4}{\pi^2(y^2-1)}\ln(y)\ ; y > 0$$

So, $P(0 < Y < 1) = \int_0^1 \frac{4}{\pi^2(y^2-1)}\ln(y)\,dy$ .....(i)

Now $P(0 < Y < 1) = P(0 < X_1 < X_2) = P(X_1 < X_2) = \frac{1}{2}$.....(ii)
As X_1, X_2 are two i.i.d random variables.

Equating (i) and (ii) we get,

$$\int_0^1 \frac{4}{\pi^2(y^2-1)}\ln(y)\,dy = \frac{1}{2}$$

So, $\int_0^1 \frac{1}{(y^2-1)} \ln(y)\, dy = \frac{\pi^2}{8}$ .....(iii)
Since, $0 < y < 1$ we will expand the series

$$\frac{1}{(y^2-1)} = -\sum_{n=0}^{\infty} y^{2n}$$

By interchanging summation and integration, we get

$$\int_0^1 \frac{1}{(y^2-1)} \ln(y)\, dy = -\sum_{n=0}^{\infty} \int_0^1 \ln(y) y^{2n}\, dy$$

Hence ,$-\sum_{n=0}^{\infty} \int_0^1 \ln(y) y^{2n}\, dy = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$[By using integration by parts] .....(iv)
So, combining (iii) and (iv) we have

$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$

Now,$\sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} + \left(\frac{1}{4} \sum_{n=1}^{\infty} \frac{1}{n^2}\right)$

Finally , $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{4}{3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{6}$

Hence, the proof.

Conclusion

In this way we have proposed here a 'Probabilistic Method' to provide an interesting solution to the 'Basel Problem' and it is really surprising that we cannot give any deep insight about this method. This solution depicts how beautiful the 'Probabilistic Method' can be and hence many solved problems are still 'Active' in terms of giving a Probabilistic solution to the problem.

Newsvendor Problem:
Alternative Optimality Criteria

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Introducing the Newsvendor Problem: An Approach to Optimality

- The Newsvendor model deals with the problem of determining the optimal order quantity of newspapers for a newspaper vendor keeping the demand of newspapers to be random.
- Since the demand is random, the newspaper vendor incurs either a shortage cost or an excess cost depending on the quantity of newspapers sold.

The above problem plays a very significant role in the history of inventory management.

In this paper we would be discussing the various optimality criteria to determine the optimal quantity of newspapers, i.e., studying over and above the mean minimizing solution of the total cost to determine the optimum level of newspapers.

The Model

Let X: Demand.
q: Order Quantity.
c_2: Excess Cost.
Y_q: Total Cost.
c_1: Shortage Cost.
S: Supply.

Supply is same as the order quantity	Supply is random, depending on the order quantity
$Y_q = \begin{cases} c_1(X-q) \text{ if } X > q \\ c_2(q-X) \text{ if } X < q \end{cases}$	$Y_q = \begin{cases} c_1(X-S) \text{ if } X > S \\ c_2(S-X) \text{ if } X < S \end{cases}$

Some Alternative Approaches

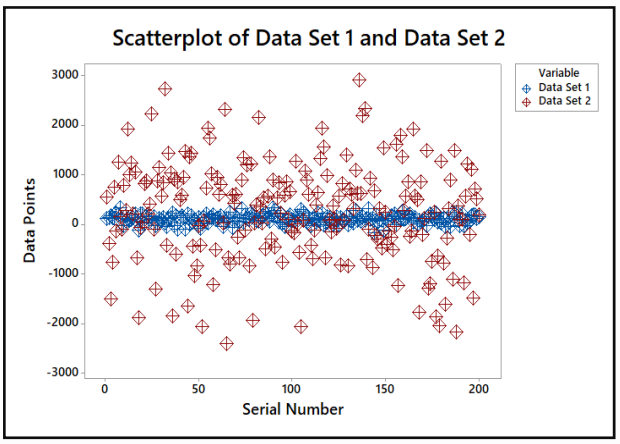
APPROACH 1:

Minimizing the Standard Deviation and Coefficient of Variation of the Cost Distribution

Application Area:

If the standard deviation of the cost distribution is large enough (as in Data Set 2), then mean minimization of the cost distribution may not give satisfactory results as the cost incurred in any particular case may be higher than the expected cost. Under this circumstance, the above approach is used.

Data Set	Mean	SD	CV
1	113.11	89.40	79.03
2	248.8	1011.3	406.50



Under Beta (2,1) distribution the variance is obtained as:

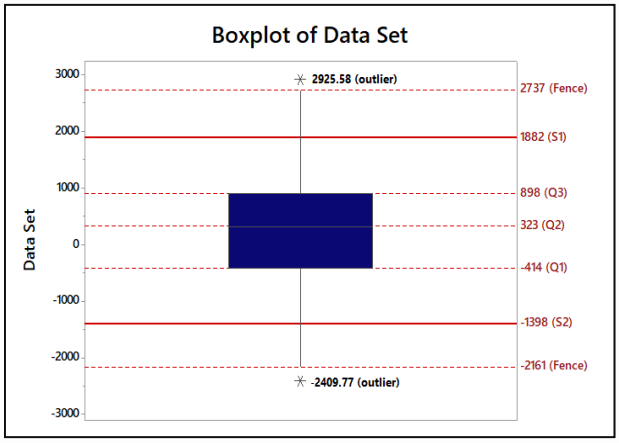
- Supply is same: $\mathrm{Var}[Y_q] = \frac{1}{18}(-3(-1+q)^3(3+q){c_1}^2 + 3q^4{c_2}^2 - 2((2-3q+q^3)c_1 + q^3c_2)^2)$
- Supply is random: $\mathrm{Var}[Y_q] = \frac{{c2}^2q^4}{18} - \frac{1}{18}{c1}^2(-9+16q-9q^2+q^4) - \left(\frac{2c2q^3}{15} + \frac{2}{15}c1(5-5q+q^3)\right)^2$

APPROACH 2:

Minimizing the Standard Deviation Penalized Mean of the Cost Distribution

Application Area:

Treating data points outside the range 'k standard deviations away from the mean' as outliers, the optimal order quantity may be obtained by this approach.
Here conventionally the data points outside the fences are considered to be outliers. Considering our problem of concern we have taken k=0.75.



Under Beta (2, 1) distribution (supply is same), S(q)=E[Yq] + k SD[Yq]
is obtained as:

$S(q) = 2c_1\left\{\frac{1}{3} - \frac{q}{2} + \frac{q^3}{6}\right\} + 2c_2\frac{q^3}{6} + k\ \{\ \frac{1}{18}\left(-3(-1+q)^3(3+q){c_1}^2 + 3q^4{c_2}^2 - 2\left((2-3q+q^3)c_1 + q^3c_2\right)^2\right)\}^{1/2}$

APPROACH 3:

Minimizing the Modal Cost

Application Area:

To ensure that the maximum possible cost is minimized we may obtain the optimal order quantity by minimizing the modal cost.
Under the demand distribution with
PDF: $f(x) = 3x^2$, $0<x<1$,
we obtain the following on minimizing the mode and equating it to 0:

- Supply is same: $q^6\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)\left(\frac{1}{{c_1}^2}+\frac{1}{{c_2}^2}\right)+q^3\left(\frac{2}{{c_1}^5}+\frac{2}{{c_1}^2{c_2}^3}+\frac{4}{{c_1}^3{c_2}^2}\right)-\frac{1}{{c_1}^5}=0.$
- Supply is random:

$$:\frac{(q^4+a^2q^2)\left(\frac{1}{c_1^2}+\frac{1}{c_2^2}\right)-\frac{q}{c_1^2}}{\left(\frac{1}{c_2^3}-\frac{1}{c_1^3}\right)(q^3+a^2q)+\frac{1}{c_1^3}}=\frac{(4q^3+2a^2q)\left(\frac{1}{c_1^2}+\frac{1}{c_2^2}\right)-\frac{1}{c_1^2}}{\left(\frac{1}{c_2^3}-\frac{1}{c_1^3}\right)(3q^2+a^2)}$$

For varying values of c1 and c2, we obtain the value of the optimal order quantity for which the mode is minimized.

APPROACH 4:

Minimizing the Probability of the Cost Distribution Exceeding a Given High Value

Application Area:

Again, minimizing the probability of the cost distribution exceeding a given high value to obtain the optimal order quantity; we may safeguard against the cost.
Under the demand distribution with
PDF: $f(x) = 12x(1-x)^2$, $0<x<1$,
we obtain the following on minimizing P(Y>y) and equating it to 0:

- Supply is same: $-\frac{36(c_1+c_2)q^2y}{c_1c_2}+\frac{12(c_1+c_2)qy(4c_1c_2+3c_1y-3c_2y)}{c_1^2c_2^2}-\frac{12y(c_1^3c_2^3-2c_1c_2^3y+c_2^3y^2+c_1^3(c_2+y)^2)}{c_1^3c_2^3}=0$
- Supply is random (supply following Uniform distribution): $-\frac{72(c_1+c_2)q^2y}{5c1c2}+\frac{6(c_1+c_2)qy(4c_1c_2+3c_1y-3c_2y)}{c1^2c2^2}-\frac{8y(c_1^2c_2^3-2c_1c_2^3y+c_2^3y^2+c_1^3(c_2+y)^2)}{c_1^3c_2^3}=0$

Solving the above equations for a given y we may obtain q.

Inference

"Uncertainty about future events is a key feature of the world we live in!"

The problem of matching demand with supply in uncertain settings is called Newsvendor Problem.
The problem is: How much to produce?
Consider the following problem at hand:
Every year Governments order flu vaccines before the flu season begins, and they make this decision before the extent or the nature of the flu strain is known.
The question is – How many vaccines to order?
To answer the question, we need to know or estimate the cost and price of the product and of course we require some data on the demand of the product. Mean minimizing the total cost to obtain the optimal order quantity is not always sufficient.
Hence keeping in mind, the problem of our concern we may apply one of the alternative optimality criteria as discussed in the paper.

A Review On the Recent Developments In Data Science

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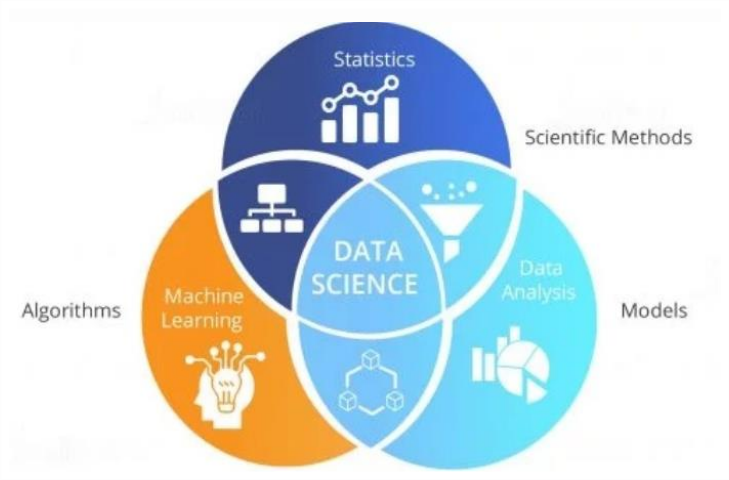
"Consumer data will be the biggest differentiator in the next two to three years. Whoever unlocks the realms of data and uses it strategically will win."

- Geoffrey Moore



Data Science, whose modern version was coined by DJ Patil (Early Data Science lead at LinkedIn) and Jeff Hammerbacher (Early Data Science lead at Facebook) in 2008, is an amalgamation of numerous tools, algorithms, and Machine Learning principles with the aim to discover hidden patterns or information from the raw data. A "Data Scientist" not only does the Exploratory Data Analysis (which is also done by a Data Analyst), but also uses Machine Learning and Advanced Algorithms to identify the occurrence of a particular event in the future. In other words, a "Data Scientist" will look at the data from many angles, sometimes angles not known earlier.

So, what is the relation between Data Science and Statistics?

Statistics is defined as the collection, analysis and interpretation of numerical data. So, if someone asks **"What kind of Statistics should a person know to become a good Data Scientist?"** The appropriate reply would be **"A person should not really worry about learning or knowing Statistics for Data Science, but rather just learn Statistics because it is actually the art of unravelling the secrets hidden inside the dataset"**. Data Scientists solve problems or help someone to take a decision, based on the available data. **They define a problem statement (by primarily asking the right questions) and then:**

1. they collect the right kind of data to perform their analysis.
2. they try to explore the data to see what it tells us.
3. they employ various techniques to infer about the data or to predict some answers for the problem statement.
4. finally, they confirm that their inferences/predictions are fairly accurate (of course, by Scientific Methods!).

To perform various tasks related to a particular query, **a Data Scientist needs to keep in mind the following algorithm:**

1. He needs to have a fair idea of the domain to which the problem statement belongs. For example, if the Data Scientist is trying to answer the question *"Why is the GDP Growth in India the slowest in a decade?"*, they should have a fair idea about GDP and Economic Status of India.

2. Secondly, except for the first step, all the other steps involve dealing with a large amount of data in digital form. **The Data Scientist should be able to get the data, cleanse it, read it, perform analytics, and employ methods to arrive at answers, in a fairly short period of time.**

All the above steps are not directly performed by a Data Scientist, but preferably from a computer, which is, in turn, instructed by a Data Scientist.

Moving on to the last section of the article, let us talk about the **relation between Data Science and Big Data Analytics!**

Big Data, coined by **Roger Magoulas of O'Reilly media in the year 2005**, points to a vast range of humongous data sets almost impossible to manage and process using traditional data management tools (which have become too obsolete)- due to their not only their size, but also their complexities. Big Data has already taken the globe by storm in the past few years, and it is of so much significance in the present date that *Geoffrey Moore, who is an American Management Consultant and also an Author quoted that, "Without big data analytics, companies are blind and deaf, wandering out into the web like deer on a freeway"*. In fact, according **to New Vantage, approximately 97.2% of organizations, spread across the world, are investing in Big Data and A.I.**

The following points depict the relationship between **"Big Data Analytics"** and **"Data Science"**:

1. Organizations require Big Data to improve productivity, figure out advanced markets, and boost competitiveness whereas Data Science provides the algorithms or mechanisms to understand and handle the potential of Big Data in a timely manner.

2. Data Science provides the methods or techniques or algorithms to decrypt or analyze data characterized by the 3Vs (Velocity Variety and Volume).

3. Data Science resorts the use of Machine Learning Algorithms and Statistical Methods to train the Computer to learn without much programming to make predictions from Big Data.

4. Data Science works on Big Data to derive useful insights or information through a predictive analysis where results or outcomes are used to make smart decisions.

The fact that Data Science not only benefits the individuals pursuing it but also the society, can be justified by the following points:

1. Redefined Customer Success: Data Science methodologies help more customer attributes to be utilized and put to use. For example, data-driven companies generally heavily depend on their audience performance to define marketing or advertising messaging in order to maximize results.

2. Numerous Sectors: In case of Agriculture Sector, according to *Matthews 2019, Data Science is completely changing the way farmers and agricultural professionals have been making decisions* which can be justified by the fact that farmers, these days, utilize this technology to decide on the amount of fertilizer, water, and other inputs that are necessary and sufficient to grow the best crop. Similarly, in case of Journalism, "Data-Driven Journalism" is considered one of the major benefits of Data Science as it heavily motivates the jobs of Journalists and the entire workflow is being driven by data. Last but not the least, Data Science benefits the Education Sector, Airline Industry, Image and Speech Recognition, Healthcare Industry etc.

Today, Data Science directly or indirectly becomes a part of our daily life! Data Science has completely transformed the way we see Data, and has already started transforming the global business landscape and it will keep doing so in a much bigger aspect in the future!

Stay tuned to deal with some "big" changes coming up!

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A Review On Matrix and Probability In Light of Graphs

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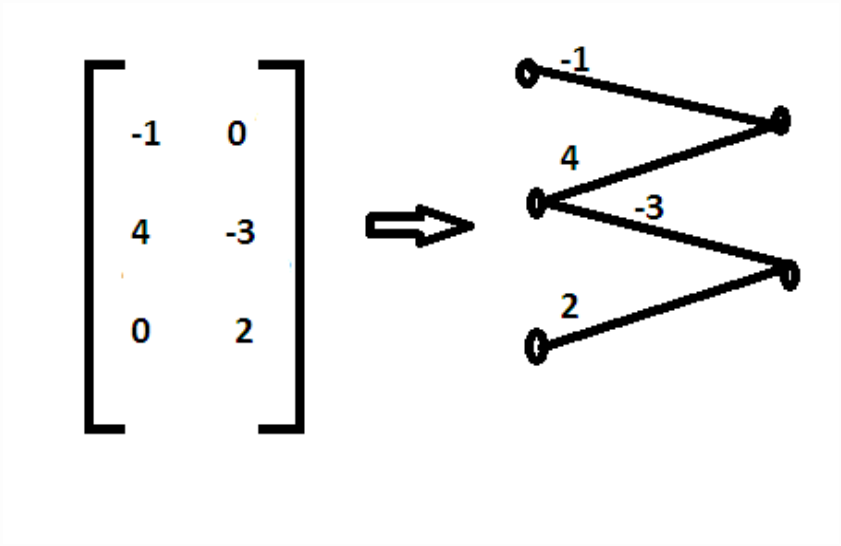
The concept of graphical representation is quite interesting to deal with. It is more interesting when we think of a matrix to be represented graphically. Let's have a glance over it:

Every Matrix Corresponds to a Graph



To be spoken briefly, Matrix can be represented as a weighted bipartite graph. Bipartite refers to the dots which come in two different types, (Here for the rows and columns) and 'weighted' refers to each edge which is marked with certain numbers.

Above is a 3×2matrix (say M) which is well depicted in graphs. For that three dots for 3 rows and two dots for 2 columns of M are drawn. And edges are drawn correspondingly for non-zero entries.

Let me try to elaborate the general set up. let, a matrix M be any array of n×m ordered members. The array can also be visualized in terms of a function

M: X × Y → R,

where,

X = $\{x_1, x_2, \ldots\ldots, x_n\}$ is a set of n elements
and
Y = $\{y_1, y_2, \ldots, y_m\}$ is a set of m elements.

Now if I intend to elaborate the matrix M to you, I need to mention each ij^{th}**entry.** Necessarily there exists a real M_{ij} for each pair of indices (i, j). In brief what role a function itself plays,

M: X × Y → R associates for every pair (x_i, y_j).Let us simply write M_{ij} for M(x_i, y_j).

A Matrix as a Function

Let a matrix M be a function. Then,

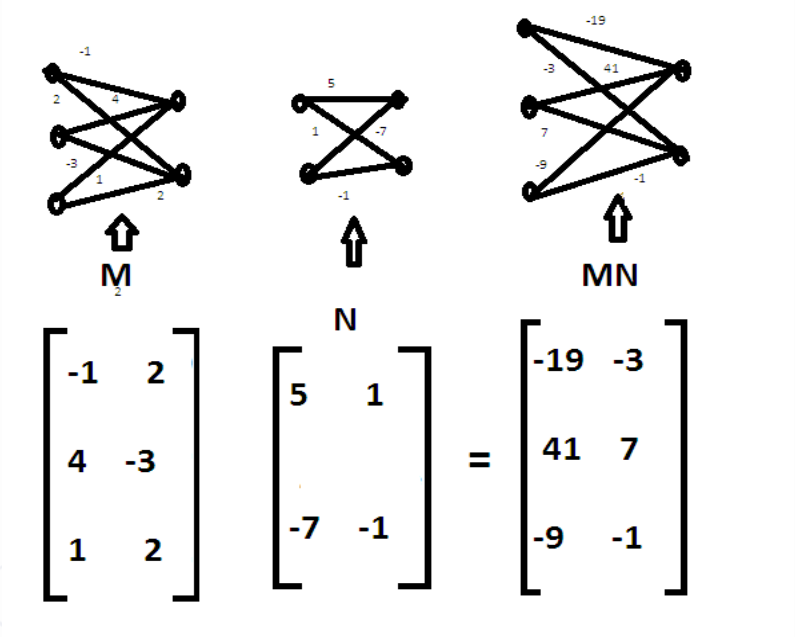
M: XxY → R

X={x1, x2, x3}
Y={y1, y2}

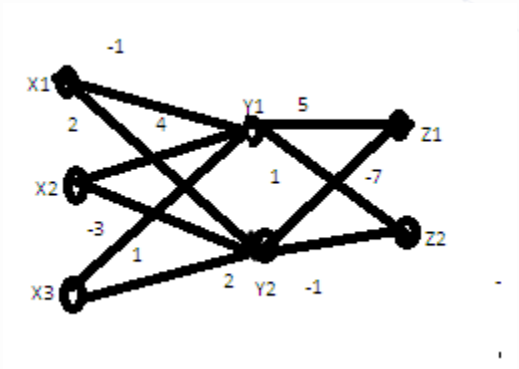
I mention we don't give lines for zero entries since it has no weightage.

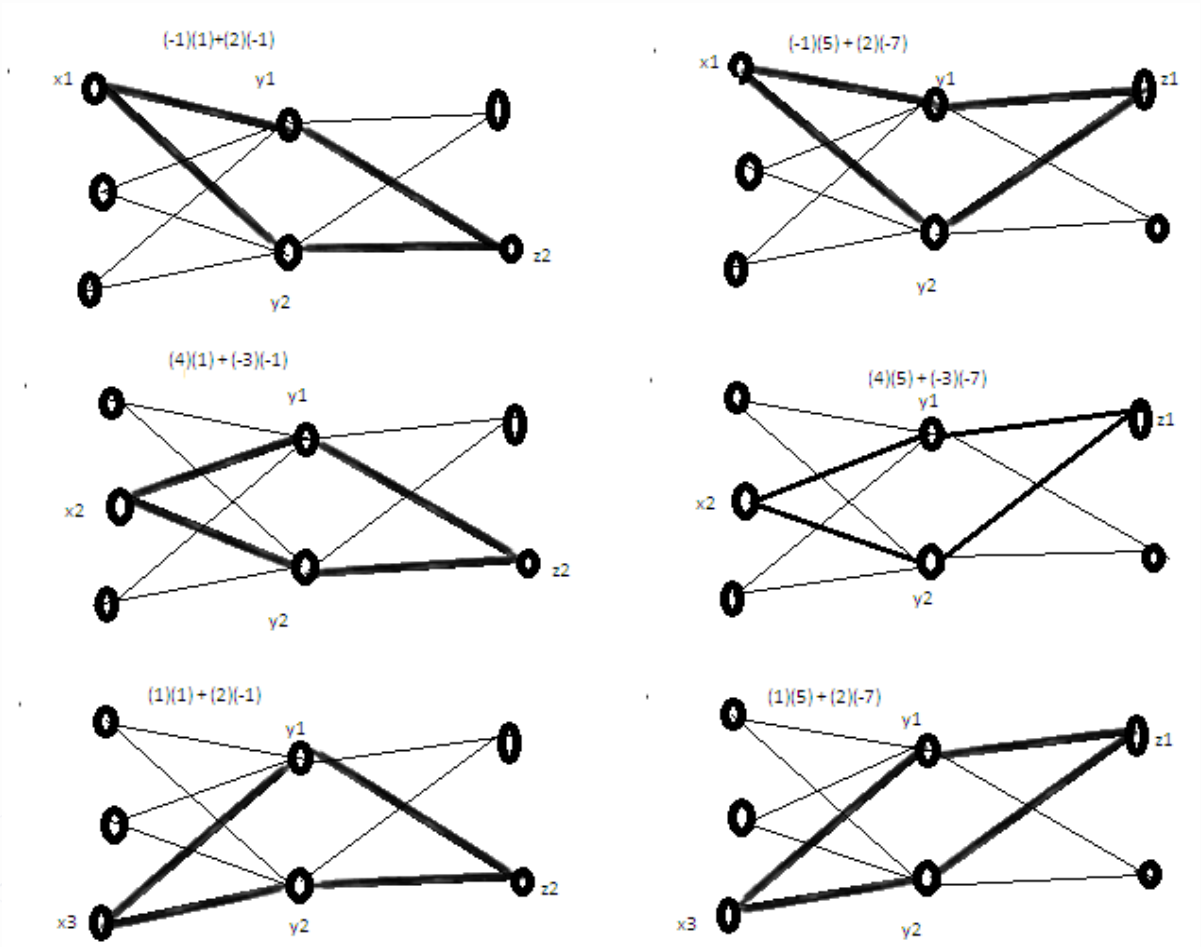
Interpretation of Matrix Multiplication

Given two matrices (graphs) M: X×Y→R and N: Y×Z→R, to multiply we can attach the two graphs side-wise travelling along paths; ij^{th}**entry** of MN – represents the value of the edge which connects x_i to z_i , and it is obtained by the product of the edges along every path from x_i to z_i and summing them. As for example:



Analysis


-1
X1
2
4
Y1
5
Z1
X2
1
-7
-3
1
Z2
X3
2
Y2
-1


(-1)(1)+(2)(-1)
x1
y1
z2
y2
(-1)(5)+(2)(-7)
x1
y1
z1
y2
(4)(1)+(-3)(-1)
y1
x2
z2
y2
(4)(5)+(-3)(-7)
y1
z1
x2
y2
(1)(1)+(2)(-1)
y1
z2
x3
y2
(1)(5)+(2)(-7)
y1
z1
x3
y2

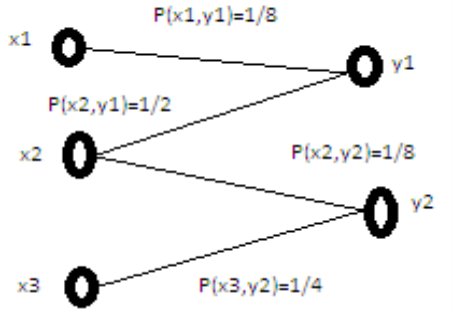
Moreover, very interesting that every probability distribution p over a product of finite sets X × Y gives rise to a matrix
whose ij[th] entry is $P(X_i, Y_j)$
i.e., the probability of (X_i, Y_j).

Let us assume a probability distribution as –

X\Y	y1	y2	Total
x1	1/8	0	1/8
x2	1/2	1/8	5/8
x3	0	1/4	1/4
Total	5/8	3/8	1

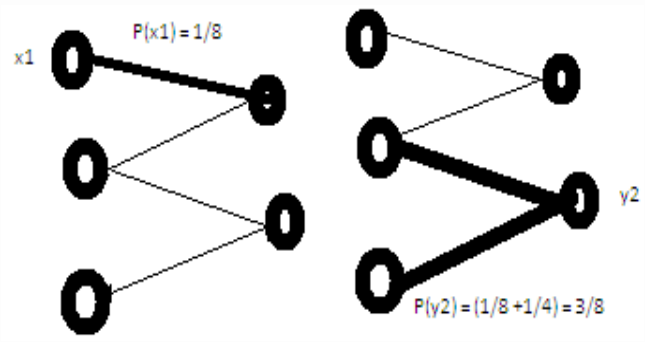
Joint Probability Distribution

On construction, joint probabilities are represented in the edges of the graphs, $P(X_i, Y_j)$ is marked as on the edge connecting them.



Marginal Probability Distribution

It is calculated by adding over rows or columns of the matrix. As for example, probability of
X_1 is P (X_1) = $P(X_1, Y_1) + P(X_1, Y_2) = \frac{1}{8} + 0$ represents the sum of the first row. Similarly the probability of Y_2 is P (Y_2) = P (X_1, Y_2) + $P(X_2, Y_2)$ + $P(X_3, Y_2) = \frac{1}{4} + 0$ represents the sum of the second column. Graphically, the marginal probabilities of X_i 's are the sum of the edges which have X_i 's as vertices. Proceeding in the same way, the marginal probabilities of Y_j are the summation over the edges containing Y_j as vertices.
Here I provide two instances:



It is very interesting that proceeding in this way we can visualize graphical interpretation of conditional probability also.

But to be stated clearly, we can do it only for discrete probability distribution and not for continuous distributions as in those cases probability at a certain point is always zero.

Moreover, bipartite graphs are well applicable in composite relations and functions also. It is also well applicable in combinatorics involved in a certain training algorithm.

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Statistics:
A Musical Viewpoint

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The thought of amalgamating science and music has been a point of debate for several years. The composer Pierre Boulez (1925-2016) said of this: "It is treason to mix the two, as science progresses but music does not". Several philosophers and critics believe that music is a spontaneous form of art which provides entertainment and that giving it scientific treatment would suck the life out of it. There are several detractors to this statement as well. The twentieth century has time and again proved that it is possible to produce sophisticated music with a technical approach.

It has been found that there exist several pieces of music which adhere to principles in physics and mathematics without the composer realizing their integration into their music. This is generally common in the music of the "Period of Common Practice" (early 1600s-1900 and slightly after). Two immediate exceptions from this period come to mind:

1. Erik Satie (1866-1925) used the Golden Ratio (two quantities are in the golden ratio if their ratio is the same as the ratio of their sum to the larger of the two quantities, numerically equal to 1.66) to proportion sections of music in his "Sonneries de la Rose + Croix".

2. Bela Bartok (1881-1945) used the Fibonacci sequence (if the 1st term of the sequence is 0 and the 2nd term of the sequence is 1, then the general term of the sequence is given by $t_n = t_{n-1} + t_{n-2}$ with $n \in N$) to fashion rhythms in his "Music for Strings, Percussion and Celesta".

That however, has not been the case post the Second World War. There exist works in which the composer constructs a piece using a particular mathematical model. Pieces belonging to this category have gained much notoriety due to their non compliance with tradition. It can produce anomalous "anti-musical" and

dadaist effects which has earned it the derision and wrath of many a listener. However it is always interesting to analyze and perceive music from a new viewpoint. This gives it freshness and a certain vigor which makes the act of listening exciting. Thus, the main intention of this article is to throw light on the development of statistical methods which are being used in the interpretation of music.

Analysis of Musical Parameters

The most preliminary way of interpreting music is by means of **graphical presentation**. Musical data is nothing but **Time Series Data**. Pitches, dynamics and timbre are some of the parameters which vary as time passes by. A line diagram can be used for presentation purposes, with time being represented horizontally from left to right and pitches being represented vertically. Due to a large dispersion in frequencies, the vertical axis can be taken in a logarithmic fashion. This will hence indicate relative changes in pitch instead of absolute ones (such a representation is called ratio / semi-logarithmic chart). The horizontal axis can assume a certain rhythmic or temporal value. This can greatly help in deducing melodic and contrapuntal (motion of several independent musical lines) structures. This also takes into account rhythmic pulse, another important parameter in music.

Harmonic content (vertical alignment of music which gives rise to chords, progressions, etc.) can be studied by **assigning numerical facts and figures** to individual pitches. This is one of the most elementary concepts of **musical set theory**. Such data can be subjected to statistical analysis, thus bringing out hidden analogies in music. Here are some recent studies which elicit my claim.

1. Nakamura and Kaneko by means of a **stochastic model** could explain the evolution in musical style as one progressed from one time period to another.
2. Miles, Rosen and Grzywacz could analyze Harmonic surprise and preferences of popular music by using **Corpus analysis**.
3. Krumhansl using **univariate, sequential statistics** could geometrically represent tonalities as the surface of a toroid, highlighting intervallic relationships like the cycle of fifths, relative minor/major and tonic triads.
4. Temperly and Clerq used **theoretical distributions** to explain and introduce harmonic concepts in rock music, like local harmonic field.
5. David Schiff analyzed the music (using **elementary descriptive procedures**) of Elliott Carter (1908-2012) to show that his "Night Fantasies" contains an all-interval twelve tone row (an array of pitches which can be used in musical construction; such composition is called serial composition and was very popular in the twentieth century)

.

Composition

Besides analysis, composition using statistical ideas has also been a common point of interest. The work of Iannis Xenakis (1922-2001) has been groundbreaking in such regard. His work "*Pithoprakta*" (composed in 1955) is his most elaborate composition using statistical techniques. *Pithoprakta* , written for strings, trombones, xylophone and wood block, translates to "actions through probability" The main impetus behind

composition was **Bernoulli's Law of Large numbers** (According to the law, the average of the results obtained from a large number of trials should be close to the expected value, and will tend to become closer to the expected value as more trials are performed) and the **statistical mechanics of gases**. In order to give this thought a musical existence, he used the **Maxwell-Boltzmann distribution** to assign each instrument as a gas molecule which freely glides through space in time. He **randomly sampled** certain values from the theoretical distribution by imagining certain limitations in temperature and pressure and then **plotted them in a two-dimensional graph** with the y-axis showing pitch (0.25 cm= 1 semitone) and the x-axis showing time (5 cm= 1 measure with the tempo quarter note equals 26). Due to this **stochastic approach to composition**, each part goes nowhere but the whole mass of the music follows Bernoulli's law.

The above stated technique is in general cumbersome. However, computer generated algorithms are now using **Markovian Chains** and **pattern recognition** to form melodies. These chains can not only produce randomly generated melodies, but at the same time can statistically analyze pitches and rhythm to produce music reminiscent of a particular composer. An example of this was the Google Doodle released on the birthday of Johann Sebastian Bach (1685-1750) which used similar processes (along with patches of AI and ML) to provide perfect four-part harmony in the style of Bach to any randomly inputted melody. Recent studies use data science in order to analyze the String Quartets of Beethoven (1770-1827). The use of **fractals** in the music of Gyorgy Ligeti (1926-2006) has also contributed to a statistical development in music.

Conclusion

Despite its currently small scale, more and more studies are being conducted which aim at developing statistical models in composition and analysis. This has led to an increased awareness in the various procedures in music and thus bringing forward music to a larger audience. The periodic progress of such approaches is inculcating new ways of perceiving music. Music can organically evolve as a result of such efforts, thus rendering it revolutionary and making every listener and composer susceptible to innovative, original influences and objectives.

SUGGESTED LISTNING:

1. **Satie: Sonneries de la Rose + Croix.**
2. **Bartok: Music for Strings, Percussion and Celesta; Sonata for two pianos and percussion.**
3. **Carter: Night Fantasies**
4. **Ligeti: Clocks and Clouds**
5. **Xenakis: Pithoprakta; Metastaseis**

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4. **Statistics, Structure and Style in Music: Carol Lynne Krumhansl**
5. **Statistical Analysis of Harmony and Melody in Rock Music: Temperly and Clerq**
6. **The Music of Elliott Carter: David Schiff**
7. **Formalized Music: Thoughts and Mathematics in Composition: Iannis Xenakis**
8. **Decoding Beethoven's music style using data science: Ecole Polytechnique Fédérale de Lausanne**

From Newspapers to Airline Tickets:
Approaching Optimality With the Newsvendor Model

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Production Inventories and Classical Newsboy Problem

Inventory refers to the stock of goods and materials that a business holds with the aim of selling those.
Uncertainty about future events is a key feature of the world we live in! The problem is how much to produce? This question plays a vital role in managing the inventories meticulously.
Be it a rice production factory, car rental company or the Airlines each and every one needs to manage and regulate the amount of goods they produce that is producing an optimal quantity which would minimize the cost incurred and maximize the revenues earned by the respective companies.
The simplicity of the Classical Newsboy Problem stretches its applicability beyond imagination. It formulates the framework of Inventory Analysis determining the optimal level which is to be produced so that the total expected cost gets minimized and the profit earned is more.

Introducing the Classical Newsboy Problem:

"Distributing newspapers in the morning...
Noble but monotonic right?"

this is what newsboys start their day with.
Classical Newsboy Problem accepts the monotonicity of the job of newsboys but offers great profits. In this practical situation; newsboys has a certain amount of newspapers with them (say q) ; the demand(X) of newspapers being random; as a result of which, at the end of the day he might face shortage or may be left with some excess newspapers in his hand. Accordingly, he has to incur shortage or excess cost. The demand being random, naturally, the cost incurred will also be a random variable.

In the following problem proposed considering two cases: supply of newspapers- when it is same and varies with respect to order quantity, we derive the optimal order quantity using the cost function for different probability distributions which the demand follows; by mean minimizing technique and thus finding out total optimal cost.

The Model

X→DEMAND
q→ ORDERED QUANTITY
f(x)→PDF OF THE DEMAND RANDOM VARIABLE X
c_1→SHORTAGE COST PER UNIT SHORT
c_2→EXCESS COST PER UNIT EXCESS

When Supply varies around the ordered quantity of newspapers.

S→SUPPLY
S~R(q - a , q + a)
The PDF of S is given by,

$$g(s)=\begin{cases}\frac{1}{2a}, & \textbf{if } q-a<s<q+a\\ 0, & \textbf{otherwise}\end{cases}$$

When Supply is same as the ordered quantity of newspapers	*When Supply varies uniformly around the ordered quantity of newspapers*
COST FUNCTION $C(q)=\begin{cases}c_1(X-q), if\ X>q\\ c_2(q-X), if\ X<q\end{cases}$ *EXPECTED TOTAL COST* $\Rightarrow T(q)=E[C(q)]=\int_q^\infty c_1(x-q)f(x)dx + \int_0^q c_2(q-x)f(x)dx$ The objective is to obtain 'q' by minimizing T(q) (the expected total cost) i.e. by equating *T'(q) to 0.*	*COST FUNCTION* $C(q)=\begin{cases}c_1(X-S), if\ X>S\\ c_2(S-X), if\ X<S\end{cases}$ *EXPECTED TOTAL COST* $\Rightarrow T(q)=E[C(q)]=\int_{q-a}^{q+a}\int_s^\infty c_1(x-s)f(x)g(s)dxds + \int_{q-a}^{q+a}\int_0^s c_2(s-x)f(x)g(s)dxds$ To obtain 'q' by minimizing T(q) (the expected total cost), we take the equation *T'(q)=0.*

Mathematical Framework

Demand Distribution of finite range i.e.,

Beta(1,2) Distribution [X~B(1,2)]

The PDF is given by,

$$f(x)=\begin{cases}2(1-x), 0<x<1\\ 0, \text{otherwise}\end{cases}$$

Situation 1

Let us consider the demand follows a B(1,2) distribution under the situation of same supply:

The Expected Total Cost is given by,

$$\Rightarrow T(q)=E[C(q)]=\int_q^1 c_1(x-q)2(1-x)dx + \int_0^q c_2(q-x)2(1-x)dx$$

$$\Rightarrow T(q)= 2c_1 \int_q^1 (x-x^2-q+qx)dx + 2c_2\int_0^q (q-qx-x+x^2)dx$$

$$\Rightarrow T(q) =\frac{c_1}{3}\{(1-q)^3\} + \frac{c_2}{3}\{q^2(3-q)\}$$

Now to minimize the expected total cost, we take the equation T'(q)=0

$$\Rightarrow T'(q)=-c_1(1-q)^2+\frac{c_2}{3}2q(3-q)-\frac{c_2}{3}q^2=0$$

Optimal Order Quantity:

$$\Rightarrow q_{opt} =1-\sqrt{\frac{c_2}{c_1+c_2}}$$

The Expected Total Optimal Cost is then given by,

$$\Rightarrow T(q_{opt}) = \frac{c_1}{3}\{(1-q_{opt})^3\} + \frac{c_2}{3}\{{q_{opt}}^2(3-q_{opt})\}$$

Numerical Illustrations

CHOICE OF C_1 AND C_2	q_{opt}	$T(q_{opt})$
$c_1=1, c_2=2$	***0.18350***	***0.24467***
$c_1=1, c_2=0.5$	***0.42265***	***0.14088***
$c_1=1, c_2=1$	***0.29289***	***0.19526***

Situation 2

Let us consider the demand follows a B(1,2) distribution under the situation of varying supply:

The Expected Total Cost is given by,

$$\Rightarrow T(q)=E[C(q)]= \int_{q-a}^{q+a}\int_s^1 c_1(x-s)2(1-x)\frac{1}{2a}dxds + \int_{q-a}^{q+a}\int_0^s c_2(s-x)2(1-x)\frac{1}{2a}dxds$$

$$\Rightarrow T(q)= \frac{c_1}{a}\int_{q-a}^{q+a}\{\int_s^1 (x-s+sx-x^2)dx\}ds + \frac{c_2}{a}\int_{q-a}^{q+a}\{\int_0^s (s-x+x^2-sx)dx\}ds$$

$$\Rightarrow T(q)= \frac{c_1}{a}\left[\frac{a}{3}-qa\right]+\frac{c_1+c_2}{2a}\left[\frac{2}{3}(3q^2a+a^3)-\frac{2}{3}(q^3a+qa^3)\right]$$

$$\Rightarrow T(q)= \frac{c_1}{3}-qc_1+\frac{c_1+c_2}{3}(3q^2+a^2-q^3-qa^2)$$

Now to minimize the expected total cost, we equate T'(q) to 0.

$$\Rightarrow T'(q)=-c_1+\frac{c_1+c_2}{3}(6q-3q^2-a^2)=0$$

Optimal Order Quantity:

$$\Rightarrow q_{opt} = 1 - \sqrt{\frac{c_2}{c_1+c_2} - \frac{a^2}{3}}$$

The Expected Total Optimal Cost is then given by,

$$\Rightarrow T(q_{opt}) = \frac{c_1}{3} - q_{opt}c_1 + \frac{c_1+c_2}{3}\left(3{q_{opt}}^2 + a^2 - {q_{opt}}^3 - q_{opt}a^2\right)$$

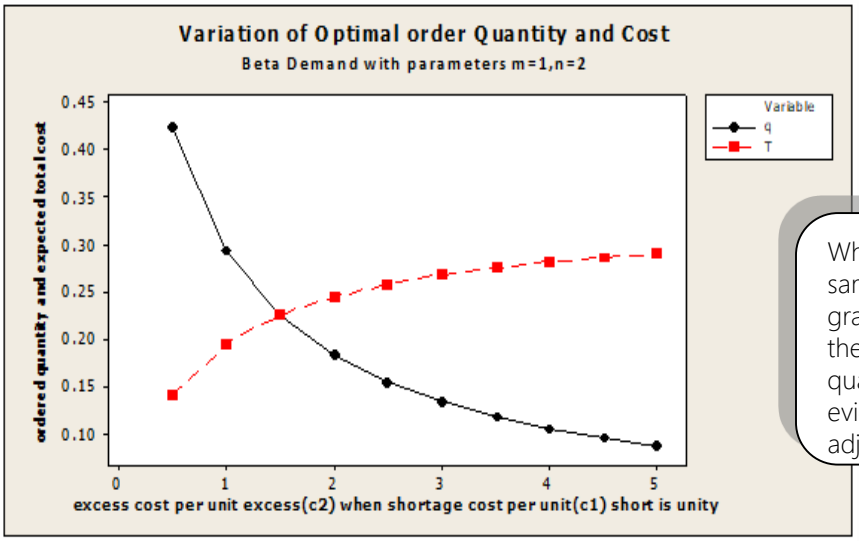
Numerical Illustrations (We take a=1)

CHOICE OF C_1 AND C_2	q_{opt}	$T(q_{opt})$
$c_1 = 1, c_2 = 2$	***0.42265***	***0.94843***
$c_1 = 1, c_2 = 0.5$	***1.00000***	***0.33333***
$c_1 = 1, c_2 = 1$	***0.59175***	***0.57594***

Graphical Illustrations

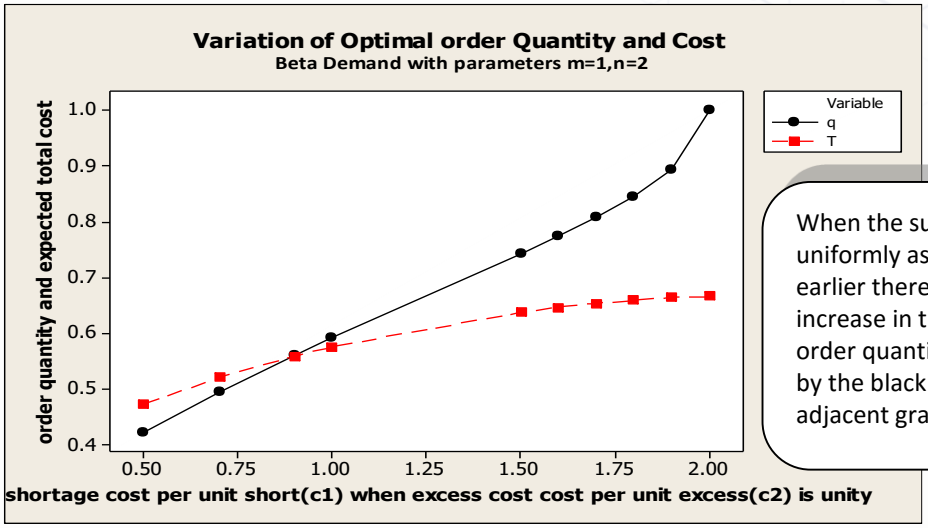
Here we consider graphical representation (variations of Optimal Order Quantity and the Optimal Cost with respect to the per unit shortage and excess cost for the demand(X) distribution) of the above Numerical situations with the following two specifications:
c1 (per unit shortage cost) is fixed at 1 and c2 (per unit excess cost) varies.
c1 (per unit shortage cost) varies and c2 (per unit excess cost) is fixed at 1.

Situation of Same Supply



When the supply is same; there is a gradual increase in the optimal order quantity which is evident from the adjacent graph.

Situation of Varying Supply



When the supply varies uniformly as discussed earlier there is a steep increase in the optimal order quantity implied by the black line in the adjacent graph.

The Newsvendor Model: Is it resourceful?

Want to know how the International Air Transport (IATA) hosted a net profit of $35.5 billion in 2019 slightly ahead of 2018's net profit of $32.3 billion!!!

From the numerical computations, we see that, the optimal order quantity and the associated cost is more when the supply is random. This excess Cost needs to be incurred for allowing the uncertainty in supply. Do these answers your queries or does it help you to understand the framework of the problem on a broader aspect? No, right?

Let's conclude with some important applications of the above problem proposed and answering the proposition considered above.

We all have heard about OVERBOOKING: which involves controlling the level of reservations to balance the potential risks of denied service against the rewards of increased sales and hiked profits.

Overview of the practical situation:

Suppose a customer books a ticket on a particular flight but eventually does not show up during the scheduled departure, the Airlines end up in a vacant seat resulting a loss in revenue.

On the other hand suppose the demands are higher than the actual number of seats available and also there are no shows of passengers to be expected, then the Airlines may deprive itself from acquiring an added profit.

Hence the Policy of Overbooking based on the exact problem proposed above centralized on the Classical Newsboy Model helps the Airlines to earn greater revenues and strengthened profits by determining the optimum level of tickets to be sold.

Is it sufficient to prove the mettle of the Newsvendor Model or is it enough to believe in the simplicity of the model which it offers? If, not then we can prove it, because not only the model formulates the policy of overbooking in airlines but also seeks it's applicability in "CAR RENTAL POLICY", "TICKET BOOKING PROCEDURE IN SPORTS PROGRAMMES", etc.

How Does Netflix Use Analytics To Prevent Churn

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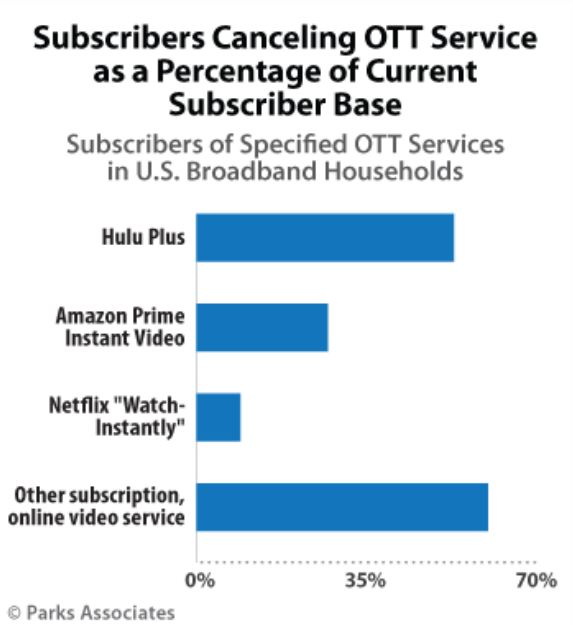
"There is only one boss. The customer. And he can fire everybody in the company from the chairman on down, simply by spending his money somewhere else."

-Sam Walton

Companies spend most of their time and effort in acquiring new customers. However, the cost of retaining a customer is 5 times cheaper than acquiring a new one. Customer retention is the method of analysing how many companies continue to buy from you and are therefore loyal to your brand. Churn is on the other end of the spectrum. It's the method of finding out how many customers stop buying from your company.



With the coming up of multiple new video streaming platforms like Amazon Prime Video, Hulu, HBO, Hotstar, etc, it is becoming very difficult for Subscription Video On Demand (SVOD) companies to retain its customer base. Netflix is the only SVOD company having positive retention. Rivals like Amazon Prime Video, Hulu, AT&T owned by HBO now are facing a higher churn rate than Netflix. While they are spending a lot

of their resources on acquiring new customers, they don't do anything to stop the Churn rate once the free trial is over. Let us see how, amidst rising competition, is Netflix preventing Churn.

Netflix uses a strategy called Customer Journey Analytics.

A third of the US households have an active Netflix subscription. Recent research by Parks Associates showed that only 4% of U.S. broadband households cancelled their Netflix service — representing almost 9% of Netflix's subscriber base. Netflix achieved this by using a strategy called Customer Journey Analytics.

What is Customer Journey Analytics?

Customer journey analytics is the process that companies use to understand users at an individual and personal level in order to give them the best user experience. They do this by analysing the user's interaction with the company through various channels. Using the data collected, they try to understand the preferences of the user.

How is Netflix using this process to retain customers?

Netflix asks its users if it wants to continue with the subscription at the end of every month. It is very easy to lose customers this way. This option given to the users makes it even more crucial for Netflix to ensure that its users remain loyal to them. They ensure that the users don't only use Netflix for a trial period but subscribe and actively continue using it.

Netflix ensures to provide the users with a wide variety of shows and movies to keep them hooked. But apart from this, Netflix also studies the users very carefully. These are few of the things they collect data on:

- Which shows you watch
- If you finish the entire season or not
- Where you pause, fast forward, rewind, etc
- If you leave an episode midway, how long does it take for you to come back and restart it? If you even restart it.
- How long does it take for you to complete an entire season
- Which day of the week do you watch the shows
- There are speculations that it also collects data on brightness, volume level, movie settings, etc

Using these, they try to create a personalized experience for you. They recommend you shows based on your watch history. The trailers of these new shows are also personalized just for you. For example, when House of Cards was released on Netflix, different viewers were exposed to different trailers. If your search history showed that you were a David Fincher fan (the director of House of Cards), trailers portraying his best-directing skills were shown to you. If you had viewed films with Kevin Spacey, then your trailer centered around him.

Well, this is for the users who are already actively watching Netflix.

What about those who have stopped watching Netflix?

Email notifications:

Users are notified about new shows being added, depending on your watch history,

through your emails. These emails are specially designed. They don't just inform you about the show, but also give you an option to "watch it now" or "add it to your playlist". This helps in direct and fast conversion of users who weren't active on Netflix, to open the app/site and watch the series.

Push notifications:

No one usually likes push notifications. To get the attention of the user, the push notifications are designed in a very simple and straightforward way. It informs you of a new season of a show being added (depending upon your watch history).

Every user is reluctant to changes made on the app. Humans, in general, resist changes. Every new feature added is first tested using split testing where Netflix randomly selects 300,000 users from around the world and introduces this feature to them. The reactions are then studied. Netflix also gives every user a demo every time a new feature gets added so that they become comfortable with the changes.

Whenever a show is recommended to you, Netflix has only 90 seconds to convince you to start watching it. Their trailer needs to be as personalized for you as possible.

Even still, with so many options to keep users informed across nearly every type of device, Netflix is continuing to test, innovate and refine its algorithms to prevent churn and keep users watching — and those users are at its core in a quest for never-ending user experience growth.

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Expecting the Unexpected

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If you are in the statistics department and you are highly satisfied after choosing statistics you are more likely to prefer back gate over front gate. Sounds very "nonsensical" isn't it ? This is the world of spurious correlation. Our objectives to have a peep into this funny World.

In real life we often find variables which are expected to be uncorrelated but on measuring they are found to be correlated which makes no sense. These types of correlation are called Spurious Correlation or nonsense correlation which leads to serious interpretation fallacy. One thing that should be noted is that correlation doesn't imply causation. If two variables are correlated doesn't imply that one has caused the other variable to change.

There may be a pair of data which are affected in the same or opposite manner by a third hidden variable called the "Lurking Variable".

Spurious Correlation: A Few Interesting Examples

Spurious correlations are often very obvious and are thus often unnoticed by most people for example number of cars on road v/s infant mortality rate. Here time is the lurking variable which is the most common known lurking variable since time progresses number of cars increases and infant mortality decreases Nonsense correlation can be observed between bivariate data as well as other kinds of data.

For eg:
❑ If we consider a group of children between the age 5-15, then we will find an unexpected correlation between the two variables
X: Shoe Size Y: Marks in a handwriting competition,
Lurking Variable : Age

❑ X: Time needed to cook
Y: Tendency to have breathing problems
Lurking Variable: Altitude of place

Spurious Correlation In Our STSA Department

We thought to find out some of the hidden correlations that could be found in our department, so for it a questionnaire was formed. Data collection:

Google forms were forwarded to all students of STSA Departments
- Name, Roll Number, Semester
- Total credit hours collected
- On a scale of 1 to 5 how much are you satisfied after choosing Statistics
- On a scale of 1 to 5 how much do you prefer using back gate over front gate
- Hours devoted per day for competitive exam preparation
- % of hours (Out of total study hours) devoted to study General Elective(GE)

Forms were forwarded via social media The response rate was moderate.
The data was copied to minitab and the following scatterplots were drawn and observed

(i) Credit hours v/s time devoted for competitive exam preparations In this case, we found that students with more credit points, invested more time for competitive exam preparation.
(ii) Credit hours v/s time devoted to learn GE. In this case, we found that as credit hours increased, students tended to invest more time for studying GE subjects.
(iii) Satisfied after choosing statistics and preference of back gate. As stated in the beginning, those who are highly satisfied after choosing statistics, are more likely to prefer back gate over front gate.

All the above correlations may logically seem unreal but few of them are rather very obvious. As mentioned earlier, correlation doesn't imply causation. So we should not draw absurd conclusions such as one has caused the other variable to change. These correlations are just a consequence of the impact of a third lurking variable.

Correlation (i) and (ii) is the outcome of the third variable: The year in which the student studies.

As the year of a student progresses:
1) Number of credit hours collected increases
2) Time devoted for competitive exam preparation increases.
3) Time devoted to study GE decreases(since 1st year students tend to devote more time in studying maths than 2nd year students do in studying comp/eco. Also 3rd year students don't have GE at all.)

One thing is to be revealed. We had mentioned in the google forms that 5 people among the ones whose both the answers will be the same will be given a gift. This influenced not all but many of the respondents to answer in that manner. Thus on obtaining the scatterplot the data points tended to cluster around the line y=x. This gave birth to the absurd positive linear relationship given in (iii) with the lurking variable being none other than Us: The framers of the question. Here we made a few assumptions that the variables were not originally correlated and it is entirely our manipulation that caused the correlation.

So in this article, we intended to illustrate how widespread nonsense correlation is but is often left unnoticed in practice. We tried to

demonstrate how there may be much more complicated cases than just cause-effect relationships. Correlations may be caused by a third hidden FACTOR and thus illogical correlations should not mislead us to draw false conclusions. We would like to end this article by quoting a paragraph from Darrell Huff's well known classic 'How to lie with Statistics'

"Permitting statistical treatment and the hypnotic presence of numbers and decimal points to befog causal relationships is little better than superstition. And it is often more seriously misleading. It is rather like the conviction among the people of the New Hebrides that body lice produce good health. Observation over the centuries had taught them that people in good health usually had lice and sick people very often did not. The observation itself was accurate and sound, as observations made informally over the years surprisingly often are. Not so much can be said for the conclusion to which these primitive people came from their evidence: Lice make a man healthy. Everybody should have them. As we have already noted, scantier evidence than this— treated in the statistical mill until common sense could no longer penetrate to it—has made many a medical fortune and many a medical article in magazines, including professional ones. More sophisticated observers finally got things straightened out in the New Hebrides. As it turned out, almost everybody in those circles had lice most of the time. It was, you might say, the normal condition of man. When, however' anyone took a fever (quite possibly carried to him by those same lice) and his body became too hot for comfortable habitation, the lice left. There you have cause and effect altogether confusingly distorted, reversed, and intermingled."

REFERENCE:

How to lie with Statistics, Darrell Huff

Uses of Graphs In EDA

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The world is based on visualization. Visualizing something, we understand the facts behind or gain some knowledge from it. If we face some problems due to some issues, we try to find the solution for that and when the problem is solved if the situation comes again in future, we solve the problems using past experience. Similarly, while handling a dataset, we need to know the characteristics of the data, How the data behaves, what are the features?

- Is it large?
- What is the tendency?
- How scattered the data is?
- Is there any unusual points or outliers?
- What is the relationship or association between variables? etc.

And the need varies from one dataset to another. Although there are some statistical measures for that but data visualization is the better method to comprehend what data reveals to us. We cannot apply any statistical method for analyzing a dataset, before we know it well. For example, we cannot use moment measures i.e., mean as a measure of central tendency or standard deviation as a measure of dispersion without knowing that, whether the data is extremely skewed or it contains outliers. It is due to non-robustness and non-resistance of mean and standard deviation.

Thus, it is primary tusk to visualize data after collecting it. According to John W. Tukey "Three of the main strategies of data analysis are:

1. graphical presentation.
2. provision of flexibility in viewpoint and in facilities,
3. intensive search for parsimony and simplicity." and he introduced this part of descriptive statistics as Exploratory Data Analysis (EDA).

Here our main concern is to show how interestingly graphical methods helps us to get insight of the data. In EDA techniques the data is simply visualized, plotted, manipulated, without any assumptions, in order to help assessing the quality of the data and building models. Throughout the EDA four main themes appear and often combine. These are resistance, residuals, re-expression, revelation.

Actually, EDA techniques are cross classified as,

1.Univariate (only one variable, exposure or outcome),

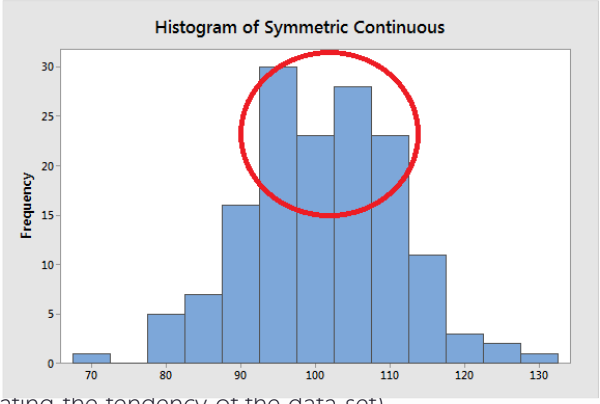
2. Multivariate (several exposure variables alone or with an outcome variable) methods.

Univariate graphical EDA techniques

There are several graphical methods under univariate graphical EDA. Like – Histogram, stem and leaf display, Boxplot, 2D plots etc. We will discuss them one by one.
Histograms are among the most useful EDA techniques, and allow us to gain
insight into data, including distribution, central tendency, spread and outliers. Histograms are bar plots of counts versus subgroups of an exposure variable. It helps us to find the outliers. It helps us to confirm that an operation on a dataset is successful (like log transformation, which is sometimes needed in order to make a unknown distribution to known distribution) . But Histograms are "old fashioned" (John W. Tukey). Stem and leaf Display is more intensive way to get insight of the data. It enables us to organize the numbers graphically in a way that directs our attention to various features of the data. When we work by hand, it is easier to construct, and it takes a major step in sorting the data. From that we can easily find the median and other "letter values" based on the ordered batch. Actually, it helps us to find any pattern in the data values. In diagram-01, we have presented a stem-leaf display of a Random data generated from Minitab (geometric, event probability 0.2). The 1st column represents the "Depth" of the data (defined by John W. Tukey). and 2nd and 3rd column represent the stem and leaf respectively.

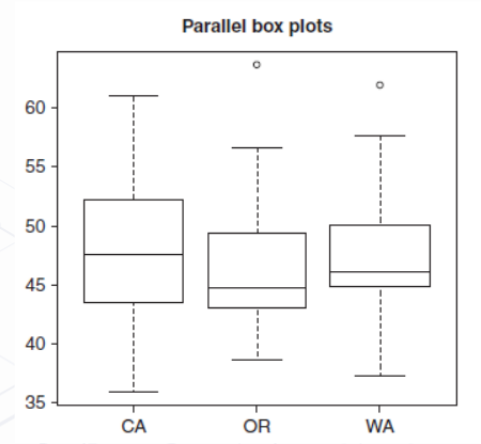
```
Leaf Unit = 0.10

 13   1   0000000000000
 23   2   0000000000
(6)   3   000000
 21   4   00000
 16   5   000
 13   6   0
 12   7   0000
 8    8   00
 6    9
 6    10  00
 4    11  0
 3    12  0
 2    13  0
 1    14
 1    15  0
```

(diagram 01)

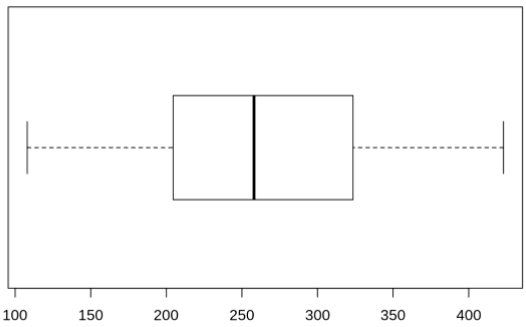


(Locating the tendency of the data set)

However, we can get a rough idea of the dataset. But before using any statistical tools we should clarify ourselves about the shape of the distribution (i.e. skewness and kurtosis). As outliers are important concern in any statistical analysis. So, we need to detect the outliers. Several non-graphical techniques are used like- Tukey's Method, Z-score, Modified Z-score etc. But graphical methods have always an extra advantage over non-graphical technique. And the method here is Boxplot (or, also known as five point summery). It is another type of EDA technique which gives the information about central tendency, dispersion, skewness, outliers. But they can hide some aspects of a dataset like multimodality. Boxplot is an excellent EDA technique because it relies on robust statistics like median and IQR. If the data point lies between the lower fence & outer fence, it is said to be 'suspected outlier' and if lies outside the upper fence, it is called 'potential outlier'(sometimes 'Boxplot outlier'). Actually, one drawback is that, though Boxplot is better EDA technique, But the number of outliers which is detected by boxplot depends on Sample size. Side-by-side boxplot is used for easy comparison between several groups. For example, consider the dataset "chickwts" in R. We can easily compare between the effectiveness of feed types on weights of chicks using boxplots side by side. Suppose, for a specific feed type, the variation (which is explained by the box area) is less and it leads to higher weights of chicks. Then, this specific feed should be used for chicks. Thus, side by side boxplots have a special importance is EDA.



(Side by side bloxplot - diagram 02)



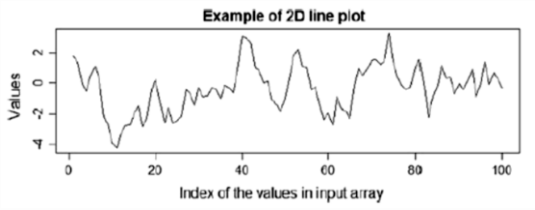
(Boxplot)

Now, we know about the skewness of the dataset and also the outliers (Although, there are also some outliers which are not detected. So, to detect them, we need to use statistical tools). There is also another graphical method called 2D-Plot(diagram-03). 2D line plots represent graphically the values of an array on the y-axis, at regular intervals on the x-axis. Actually, it reflects how typically the values of x changes & how many times it occurs.

To Identify the Distribution

When we know about different features of the data (like-tendency, spread, shape, pattern) then we should know from which statistical population it has come. Because most of statistical tests are specified by type of

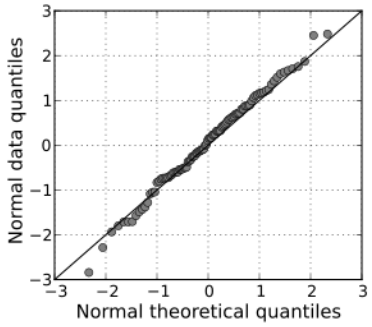
population. And to find this, we will use Probability plot. It is another type of EDA technique which is used to test whether the dataset follows a particular distribution. They are most often used for testing the normality of a data set, as many statistical tests have the assumption that the exposure variables are approximately normally distributed. Actually, we plot the theoretical quantiles vs sample quantiles. Thus, it is also called "Q vs. Q plot". The interpretation of 'Q vs. Q plot' is totally visual.



(2D line plot – diagram 03)

Deviation of the observed distribution from normal makes many powerful statistical tools useless. Thus, before applying a statistical tool to a dataset, we should know more about the dataset. The below 'Q vs. Q plot' (Diagram-04) shows the dataset perfectly follows normal distribution.

Besides the probability plots, there are many quantitative statistical tests (not graphical) for testing for normality, such as Pearson Chi-square, Shapiro-Wilk, and Kolmogorov-Smirnov.

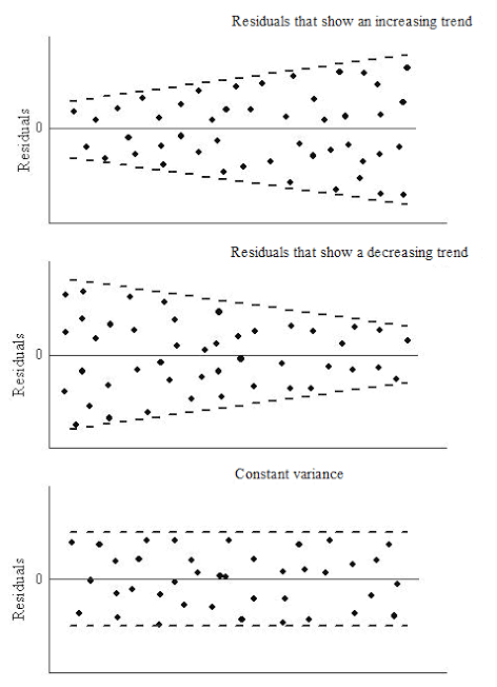


(Q vs Q Plot – diagram 04)

Multivariate graphical EDA techniques

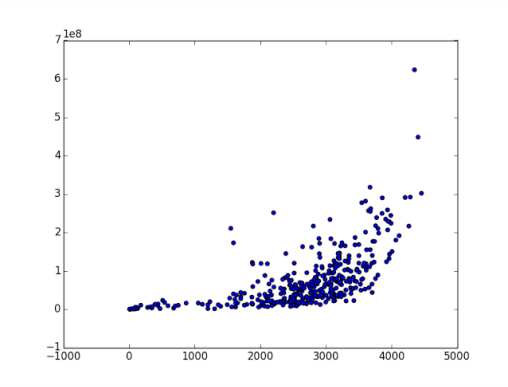
There are many multivariate graphical EDA techniques. Like- scatter plot, Bagplot, Curve fitting etc. We will discuss them one by one.

To find a relationship between two variables, one can check whether they are co-related or not. Scatter-plot is another EDA technique to know this. Scatterplots are built using two continuous, ordinal or discrete quantitative variables. Each data point's coordinate corresponds to a variable. They can be complexities to up to five dimensions using other variables by differentiating the data points' size, shape or colour. Scatterplot is used to see the type of association, the strength of association, the direction of association and presence of outliers. Also, if one asked to fit a regression model on a given data, he would start with scatter-plot. A scatterplot gives a rough-sketch about which model we should choose. After that we can



(Residual plot)

easily choose a proper regression model. For example, if we find a scatterplot like diagram-05, we should not fit a linear regression model for predicting purposes.



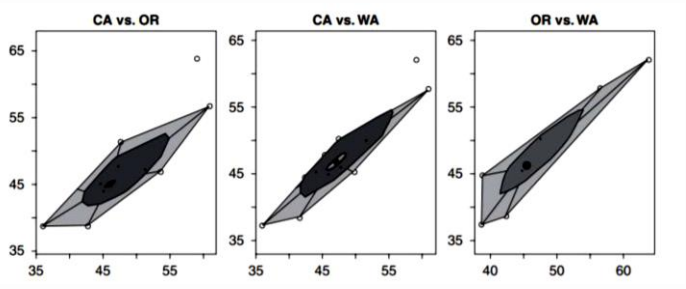
(Scatter plot: diagram 05)

Now naturally a question comes to mind, why we need to fit a curve for prediction? Ok. Fo your convenience I must say Prediction is a "ART" and the science which is used fo prediction is "Curve fitting".

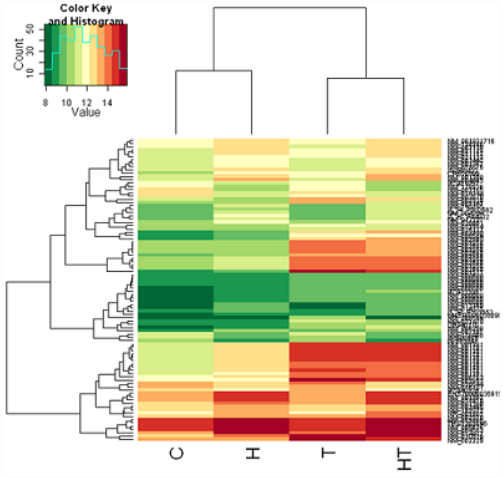
In time series analysis Curve Fitting ha importance. And to fit a curve properly fo predicting purpose, we should make a carefu examination of the residuals. It is a key attitude of EDA. This analysis can and should take advantage of the tendency of resistant analyses to provide a clear separation between dominant behaviour and unusual behaviour in the data. When the bulk of the data follows a consistent pattern, that pattern determines a resistant fit. The resistant residuals then contain any drastic departures from the pattern, as well as chance fluctuations. Unusual residuals call for a check on the details of how the corresponding observations were made and handled.

If we observe a pattern in the residual plots, then, the fitted regression model may not appropriate for predicting purposes (residual plots may also reveal some pattern due to violation of some underlying assumptions). Many real-life problems are not always explained by a straight line (because life is completely different from a straight line). In those cases, instead of using linear model we use different models (like quadratic, cubic, exponential etc.).

Now apart from this, we have Bag plot. Which is a generalization of the box plots in diagram-02. It is often a convenient way to study the scatter of bivariate data. In the construction of a bag plot, one needs a bivariate median, analogs of the quartiles, and whiskers. Tukey and his collaborators developed these. The centre of the bag plot is the Tukey median.



(Bag plot)



(Heatmap)

These are all about graphical EDA techniques. But there are also some EDA techniques like- Heatmap (to find the relationship between a set of variables), Smoother (replacement of a scatter of points by a smooth curve), Robust variant (to study both the outlier and the robust/resistant curve), Re-expression, Median polish etc.

Thus, Graphical EDA's are the essential tools in data visualization. Non-graphical tools like mean, median, mode, variance, quantile measures, Pearson measure, chi-square test, Shapiro-Wilk, Kolmogorov-Smirnov test etc. are also useful. But note that, they may give us significant results as well as useless results (wrong interpretation). Thus, graphical EDA techniques have more importance in data visualization.

After developing sufficient knowledge about the dataset, then we can apply different statistical tests and tools for analysis purpose. Actually, this part of data analysis is sometimes called Confirmatory Data Analysis (CDA).
Now to end this discussion I would like to quote something:

"as consumers of statistics, we need to act with vigilance; and as producers or explainers of statistics, we need to act with integrity"

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Importance of Normal Distribution In Aspect of Approximation

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We are interested to test whether we can treat a data set of sufficiently large size to be drawn from a normal distribution?

Here we make certain assumptions, like,
1. The data in hand is a Simple Random Sample.
2. The sample size is sufficiently large.

Let's take an simple example, Here using R software, a random sample of size 1600 is drawn from exponential (mean=5) population and we want to check if this sample can be treated as a random sample drawn from Normal population with mean=5 and sd=5. Well a question may arise about the fact that knowing in advance that the random sample drawn is from exponential(mean=5) population, there should not be any question of testing the normality. But in real life the actual population from which the data is drawn is hardly known.

Here we will apply the Kolmogorov-Smirnov test of normality to detect the normality of the data.

```
mean=5
rate=1/mean
**Taking the random sample from exponential(mean=5) population**
y=rexp(1600,rate)
**Taking the random sample from N(5,5) population**
z1=rnorm(1600,mean,mean)
**checking the normality using kolmogorov smirnov test**
ks.test(y,z1)
```

Though it's a bit obvious that based on the above test we will reject our assumption that the above data has been drawn from a N (mean=5, sd=5) population. Let's witness it objectively, i.e., we will take a look at the results obtained from the software.

```
        Two-sample Kolmogorov-Smirnov test

data:  y and z1
D = 0.16062, p-value < 2.2e-16
alternative hypothesis: two-sided
```

This clearly indicates that we have clear evidence against our assumption, that the data can be treated as random sample from N (mean=5, sd=5) population at 1% level of significance.

Now let us consider the above data and we want to test for the location(mean).

We want to test: -
H0: mean=5 against H1: not H0

Given the fact that the sample has been drawn from an Exponential distribution, we could have proceeded with an exact test of location. However here we are interested to make use of the large sample size.

Well, it is quite clear that, under the consideration that H0 is true, the data in our hand must be from, exponential(mean=5) population.

Then let us take several random samples from exp(mean=5) population with increasing sample sizes and then, for each sample, we have calculated the sample mean and subtracted it by the population mean.

Notationally, since here we took 1601 such samples, i.e., the sample sizes varying from, 400 to 2000.

Let Xi1,.....,Xij denote the random sample drawn, where, i=1,2,...,1601 and j=400,401,...2000, and i and j are ordered, i.e., (1,400),...,(1601,2000). We have calculated,

$Si = \bar{X}i - 5$,

where $\bar{X}i = 1/j \sum Xik$ j k=1

Using R, we make a visual inspection of the nature of Si's by plotting them

```
mean=5
rate=1/mean
s=array(0)
**Taking the sample sizes in an array**
n=400:2000
for(j in 1:1601)
{
s[j]=(mean(rexp(n[j],rate))-mean)
}
hist(s,xlab="plot of S_n",main="Histogram
```

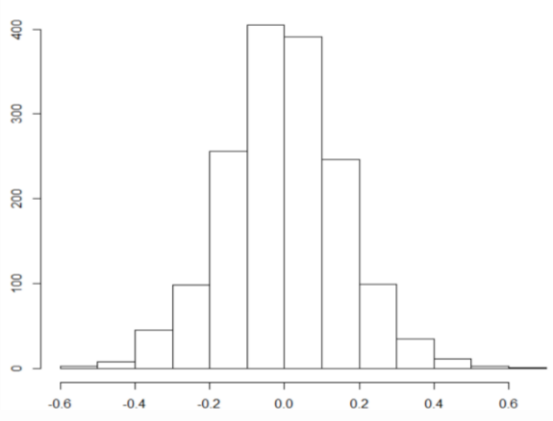
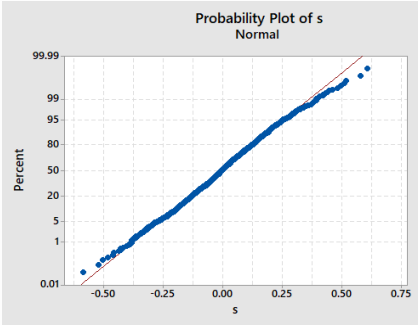


Fig.: Histogram of the Si's

We can infer that the density is symmetric and bell-shaped when plotted and most of the observations are scattered around zero and has very thin tails. To gather some approximate information about the above population, we must think about a symmetric and bell-shaped distribution which has the maximum dispersion around 0 and has very thin tails. And among the known distributions, normal distribution seems to be a plausible approximation. But this is a subjective method of selection. We now check whether the sequence {Si} can be approximated by a Normal distribution or not. Thus, we carry out a test of normality. Here, we will perform the Kolmogorov-Smirnov(K-S) test.

Since, we are clueless about the measure of dispersion, so we carry out the K-S test of the data using Minitab.



The value of the test statistic is 0.023. Hence we accept the fact that it is from the Normal population as we do not have any clear evidence against our assumption, at 1% level of significance.

Now, we test whether the population may be approximated by an N(0,1) population or not. We compare that with the help of R software.

```
        Two-sample Kolmogorov-Smirnov test

data:  s and z
D = 0.35103, p-value < 2.2e-16
alternative hypothesis: two-sided
```

At 1% level of significance we thus reject our assumption that it is from the N(0,1) population. Now, a question arises whether we should use some other symmetric and bell-shaped distribution?

Before moving further, we know that if a random variable X ~ N(μ,σ 2), then $X-\mu\ \sigma$ ~ N(0,1). Using the above fact, we redefine Sn as,

$$S_n = \frac{\sqrt{n}(\bar{X}_n - 5)}{5}$$

And the her this could be approximated by N(0,1) population.

```
> for(j in 1:1601)
+ {
+ s[j]=(mean(rexp(n[j],rate))-mean)/(mean/sqrt(n[j]))
+ }
> zl=rnorm(1601)
> ks.test(s,z)

        Two-sample Kolmogorov-Smirnov test

data:  s and z
D = 0.033104, p-value = 0.3442
alternative hypothesis: two-sided
```

failed to reject our assumption that this data can be approximated by, N(0,1) population at 1% level of significance. And for a sequence of i.i.d random variables, {Xn}, $\sqrt{n}(\bar{X}n-\mu)\ \sigma$ ~AN(0,1), where $\bar{X}\ n$ = 1 $n\ \sum\ Xi\ n\ i$=1 , E ($\bar{X}$ n)=μ and V ($\bar{X}\ n$) =σ 2 n . So we see that it may not be logical to approximate the original data by a N(0,1) population. But if the repeated samples are taken from a normal population and if 95% confidence interval is obtained for each random sample, then in the long run, 95% of these intervals will contain the mean of the density. If repeated samples are not taken from a normal population, then the confidence coefficient may not be exactly 95%, and the experimenter is usually satisfied with confidence coefficient varying from 93% to 97%. But if the deviation is quite large, then we need some other methods of testing. If in advance one knows the basic distribution of the sample, then the desired confidence interval may be found using the exact tests. But again if it is unknown, one needs statistical techniques which are applicable regardless of the form of the density. These techniques are called the nonparametric or distribution free methods.

Lindley's Paradox

A Contradicting Situation of Frequentist Approach and Bayesian Approach

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Over the decades statisticians has developed various methods to be applied in different situations to collect accurate data and analyzing it properly to derive valid and objective conclusions; all comprising a common characteristics: given the uncertainty, they try to obtain the best strategy to answer queries originated in different fields. Mathematical statistics tries to achieve this strategy using two major paradigms-

1. Classical (or Frequentist) Approach,
2. Bayesian Approach

In Classical Approach, we have considered θ, the parameter of our interest, as a fixed constant. On the other hand, in Bayesian Approach we treat θ as a random variable distributed according to PDF (PMF) $\pi(\theta)$ on Θ, the parameter space. The prior distribution $\pi(\theta)$ conveys our prior belief about the behavior of the parameter and uncertainty wrapped up in the form of a distribution.
For each $\theta \in \Theta$ and $y \in \mathcal{Y}$, the sample space $p(y|\theta)$ represents our belief that we can observe y in our study if θ is true. Once the data y is collected, we update our beliefs about the parameter θ using Bayes' theorem.
Peter D. Hoff in his book, 'A First Course in Bayesian Statistical Methods' has mentioned that 'mathematical results of Cox(1946,1961) and Savage (1954,1972) prove that if $\pi(\theta)$ and $p(y|\theta)$ represent a person's rational beliefs, then Bayes rule is an optimal method of updating the person's beliefs about θ given new information y'.

Lindley's paradox is a silent divergence between the approaches to a hypothesis testing problem that leads to contradicting conclusions for some choices of the prior distribution. It was discussed in 'Harold Jeffreys' 1939 textbook; it then became popularly known as 'Lindley's paradox' after the name of Dennis Lindley called the disagreement a paradox in a 1957 paper.

Description of the paradox

Consider that in an experiment the result is x, with two possible explanations, hypotheses H0 and H1, Lindley's paradox occurs when-

1. In frequentist approach for testing H0 ,leads to rejection of null hypothesis at 5% level of significance with a very low p-value.
2. 2. The posterior probability of H0 given x is high, indicating strong evidence that H0 is a better description of x than H1.

These contradicting results can occur at the same time.

Consider the following example:

A mountaineer asked his trainer- the route of reaching to the top of a high mountain. His trainer told him that the mountain is so steep that at each step he can only go either to the north or to the east .The trainer had reached to the top after 105000495 steps. Among them 52512259 steps were to the north and rest of the steps were to the east. Now the friend's question is:

If his steps are independent then after each step is it equally likely to go to the east or to the north?

Now, let X be the random variable denoting the number of steps to the north out of 105000495 steps.
Clearly, $X \sim Bin(n,\theta)$
Where, n = 105000495
θ denotes the probability of going to the north at each step.
Here we want to test:
$H0:\theta=0.5$ against $H1:\theta\neq0.5$

Frequentist Approach

Let f denotes the proportion of steps to the north i.e., f=X/n

The frequentist approach of testing H0 is to compute a p-value, the probability of observing a fraction at least as large as observed f= x/n=0.5001 assuming H0 is true. Because total number of steps is very large we can use Normal approximation.
We know by Central Limit Theorem,
$T= (f - \theta) / \sigma \sim N(0,1)$
Where , $\sigma2 =\theta*(1-\theta)/n$
Under $H0,\theta=0.5$
observed value of x = 52512259
n= 105000495
f=x/n=0.5001144
observed value of T, Tobs= 2.344399

Here the p-value is coming out as,
$p=P(f > 0.5001144|\theta=0.5)$
= 0.00952888

For two sided test,p-value is $2*0.00952888=0.01905776 < 0.05$
Our level of significance is 0.05.
As $Tobs > \tau0.025 =1.959964$
and p-value is less than the significance level 0.05,so,we reject H0 against H1 .

Bayesian Approach

$\pi(H0)$ denotes the prior probability of accepting H0. Similarly, $\pi(H1)$ can be defined.
Assuming no reason to favor one hypothesis over the other, the Bayesian approach would be to assign prior probabilities
$\pi(H0) =\pi(H1)=0.5$
and a uniform distribution to θ (θ being a variable its prior distribution is assumed to be uniform before drawing the sample) under H1 and then to compute the posterior probability

of H0 using Bayes' theorem,

P(H0|x) =(P(x| H0)* π(H0))/(P(x| H0)* π(H0) + P(x| H1)* π(H1))

After observing x= 52512259 steps to the north out of total steps n= 105000495, we can compute the posterior probability of each hypothesis using the probability mass function for a binomial variable,

P(x| H0) = probability of observing x when H0 is true
=nCx*(0.5)x*(0.5)n-x
=4.987215*10-06

P(x| H1) = probability of observing x when H1 is true i.e.,θ can take any value between [0,1] except 0.5
= ∫01 nCxθx(1-θ)n-xdθ = nCx * Beta(x+1,n-x+1)=1/(n+1) = 9.523765*10-09
[As in Bayesian θ is a variable ,not a fixed constant we can calculate the integral.]

From these values, using Bayes' Theorem we find the posterior probability of
P(H0|x) = probability of H0 to be true when x is observed
=0.998094,

P(H1|x)= probability of H1 to be true when x is observed
=0.001905996,

which strongly favors H0 over H1.So,here we will accept H0 against H1.

The two approaches—the Bayesian and the frequentist—appear to be in conflict, and this is the "paradox".

The Resolution of the Paradox

Lindley's paradox is encountered for a some values of moderately large sample size when H0 is very particular but H1 is more diffuse and the prior distribution chosen is not the correct one.

There are a number of reasons behind this apparently contradicting scenario where the Bayes' factor (P(H0|x) /P(H1|x)) may be very large but p-value remains fixed at a low value.

In the frequentist approach, hypothesis is tested only on the basis of the value considered under the null hypothesis and without any reference of the values under alternatives and gives answer to the question whether there is enough evidence in the dataset that is not in support of the null hypothesis so that we can reject it whereas Bayesian inference is based on the sample space and the entire parameter space considered under null and the alternative hypothesis and declares in comparison with H1, H0 is more supportive with the observations.

As H1 is much more diffuse and θ can take any value in [0,1], it leads to a very low posterior probability, P(H1|x).

Under H0, we take θ≈0.5, and ask how likely it is to see 52512259 steps to the north out of 105000495 total steps.

Under H1, we choose θ randomly from the interval [0,1], and try to find the answer of the equivalent question.

Most of the values of θ under alternative hypothesis are weakly in favour of the observations.

Actually the divergence between two approaches is not a contradiction at all, but two different measures about how two methods interprets the result of hypothesis testing:

- The frequentist method reveals that H0 is unable to describe the observed values.
- The Bayesian approach reveals that H0 is a better description for the observations than H1.

And thus the resolution of the loss, the apparent paradox.

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