Course: Discipline Specific Elective [Semester-5, Grp-A]

Semester	**5**
Paper Number	**Paper no:1 [HMTDS5011T]**
Paper Title	**Linear Programming and Game Theory**
No. of Credits	**6**
Theory/ Composite	**Theory**
No of periods assigned	**Th: 6**
Name of Faculty Member(s)	**Prof. Tarun Kumar Bandyopadhyay**
Course Description/ Objective	• Explains the role of LPP in modelling diverse types of problems in planning, routing, scheduling, assignment, and design. • Discusses basic assumptions of LPP with emphasis on linearity of constraints and objective function and discusses, through examples, the non-linear case. • Discusses how historically ideas from linear programming have inspired many of the central concepts of optimization theory, such as *duality, decomposition, convexity* and its generalizations. • Discusses economical and intelligent Simplex Method to solve LPP • Explains how LPPs always occur in maximization-minimization dual pairs and its application to Transportation Problem , Assignment problem and Game Theory • Introduces Game theory as the study of mathematical models of strategic interaction in between rational decision-makers and discusses its applications in different fields of social sciences .
Syllabus	**Linear Programming and Game Theory (78 classes)**

	Introduction to n dimensional Euclidean space E^n. Directional Derivative and direction of optimization of a linear function f:$E^2 \rightarrow E$**(3)** General form of Linear Programming Problems: standard and canonical forms. Assumptions behind Linear Programming Problems**(3)**. Graphical Solution of L.P.P and moving hyperplane method: Examples of Finite Optimal Solution; Alternative Optimal Solution; Unbounded Solution; Non-Existence of feasible solution **(3).** Linearly dependent and independent set of vectors in E^n, Spanning set, Basis, Dimension, Replacement Theorem[Proof Included]**(3)**. Elementary row and column operations on a matrix, Row rank and Column rank of a matrix and the relationship between them [statement only]. Full rank of a matrix, Rank of product of two matrices [statement only], Determination of rank of a matrix using elementary row/ column operations, Invertibility of matrices through rank **(3)**. Solution of system of linear equations: Basis matrix and Basic solution: examples. Feasible solution. Degenerate solution. Reduction of a feasible solution to a basic feasible solution with proof **(3)**. Convex Combination of finite number of points in E^n**.** Convex sets and their properties [Proof included]. Examples **(2)** Extreme points and Boundary points of a Convex set: Examples. Every extreme point of a convex set is a boundary point of the set though not conversely **(3).** Hyperplanes. Halfspaces : Closed and Open. Hyperplanes and Halfspaces are convex sets [Proof Included]. Convex set of feasible solutions of a system of linear equations and linear inequations **(2)** Directions and Extreme directions of a Convex Set. Extreme Ray. Finding Extreme directions of the convex set X= $\{\mathbf{x}: \mathbf{Ax} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$, $\mathbf{x} \in E^n$**. (2)** Polyhedral Sets and Polyhedral Cones. A point $\mathbf{x}_1$ of X= $\{\mathbf{x}: \mathbf{Ax} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$ is an extreme point of X iff $\mathbf{x}_1$ lies on some n linearly independent defining hyperplanes of X [Proof included] **(3)**

	Caratheodory's Representation Theorem of the Polyhedral Set X= {**x**: **Ax≤b**, **x≥ 0**} in terms of extreme points and extreme directions(statement only)**(3)**Correspondence between b.f.s of a system of linear equations and extreme point of the corresponding convex set of feasible region[Proof Included] :examples**(2)**. Simplex Method: Its Algebraic and Geometric Aspect **(2)** Degeneracy and Cycling. Criteria for improvement and optimality of objective function; Criterion for unbounded solution. Computational Aspect of Simplex method: Simplex table. Examples **(4)** Obtaining initial b.f.s. Artificial variable. Charne's Big M Method. Two-Phase Method **(4)** Duality Theory. Standard form of primal and dual l.p.p. dual of the dual LPP is the primal LPP (proof included). Weak duality Theorem, Fundamental Theorem on Duality(proof included) and their applications. Simultaneous solution of primal and dual l.p.p.s:examples**(4)**. Complementary Slackness Theorem (proof included) **(1)** Transportation Problem: Mathematical formulation. A balanced TP has an optimal b.f.s. In a balanced TP having m origins and n destinations (m,n≥2), number of basic variables in a b.f.s. is at most m+n-1**(2)**. Loops in Transportation Table and their properties : a subset of the columns of the coefficient matrix of a TP are linearly dependent iff the corresponding cells contain a loop(with proof)**(3)**. Obtaining initial b.f.s.: North-west corner rule and Vogel's approximation method **(3)**. Derivation of criteria of entering and departing vector and test of optimality of a balanced transportation problem **(2)**. Unbalanced transportation problem. Resolution of degeneracy**(2)** The Assignment Problem: Mathematical Formulation. Dual Assignment Problem. The Reduced Matrix. Modifying the Reduced Cost Matrix**(2).**Use of Complementary Slackness Theorem in obtaining optimal solution to an Assignment Problem. Computational Procedure**(2)**. Unbalanced assignment problem. Conversion of Maximization Assignment Problem **(2)**. Two Person zero sum game. The Saddle point and the maximin-minimax principle. Relation between maximin

	and minimax values**(2)**.Games without saddle point: Mixed strategy**(2)**. Graphical Method of solving nx2 and 2xn games **(2)**. Dominence property: generalised dominance**(1)**. Reduction of a game problem to a LPP. Fundamental Theorem of Rectangular Games **(2)**. Examples **(1)**.
Texts	(1) Linear Programming: D.J. Bhattacharya (2) Linear Programming: P.M.Karak
Reading/Reference Lists	1. Linear Programming and Network Flows: Bajara & Jarvis 2. Linear Programming: G.Hadley 3. Operations Research An Introduction: Hamdy.M. Taha
Evaluation	**CIA: 20** **End-Sem: 80**

Course: Discipline Specific Elective [Semester-5, Grp-B]

Semester	**5**
Paper Number	**Paper no:1 [HMTDS5011T]**
Paper Title	**Topology & Functional Analysis**
No. of Credits	**6**
Theory/ Composite	**Theory**
No of periods assigned	**Th: 6**
Name of Faculty Member(s)	**Prof. Rabiul Islam**
Course Description/ Objective	• Learning and application of. Topological Space as generalization of metric spaces • Study of compactness, connectedness, completeness with reference to topological spaces and their (non-) preserving nature under continuity and homeomorphism

	• Countability and separation properties with reference to topological spaces and their (non-) preserving nature under continuity and homeomorphism • Comparison between (a) linear space and normed linear space and (b) finite and infinite dimensional NLS. • Criteria of closed and bounded subsets of an n.l.s. to be compact in terms of dimensionality of the space
Syllabus	**Topology & Functional Analysis (78 classes)** **Topology** Concept of topology in a metric space: Motivation. Abstract topological space: examples, definition. Concept of basis of topological spaces. More examples, comparison of topologies **(5)**. Concept of neighbourhood, interior, exterior and boundary, open sets, closed sets, closure of a set. Subspace, product space. Closure and interior in subspace and product space**. (2).** Concept of continuous function in topological spaces. Example, definition, construction of continuous functions. Pasting Lemma. Topological equivalence, Homeomorphism, homeomorphic subsets of R, R^2, R^3 etc. (special mention on known geometric figures**).(5)** Convergence of sequence in a topological space. Non-uniqueness of limits, limit of a convergent sequence is unique in a Hausdorff space **(2**). First countabilty in context of convergence of a sequence and limit point. Heine's continuity criterion. Properties of continuous function in connection of Hausdorff space. Second countability, separability, Lindeloff properties and their interrelation. They are topological properties but some of them are hereditary and some are productive. Separable metric space is second countable(4) . Separation axioms: T0,T1,T2,T3,T3.5,T4,T5 spaces. Examples. Metric space is T5.Relation between Ti spaces(i=0,1,2,3,4,5). Regular and Normal spaces, Urysohn's lemma, Tietze's extension theorem(without proof). Metric space is normal, regular Lindeloff space is second countable**.(12)** Compactness : concept and examples, definition , [a,b] is compact in R, closed subsets of a compact space is compact. Compact subsets in R. In T2 space compact subsets are closed. Product of two compact space is

	compact, compact subsets of Rn, Heine-Borel theorem. Lebesgue covering lemma. Compactness is a topological property. Compact sets and continuous functions, FIP. In a compact space a family of closed sets having FIP has non-empty intersection. Countable compactness. Frechet compactness, first countability and other concept of compactness. Classification via compactness**.(12)** Concept of connectedness in a topological space, examples, definition, equivalent definition, it is not a hereditary property, intersection of connected sets may not be connected, union of a family of connected sets may not be connected. Closure of a connected set is connected. Connectedness is a topological property. Path connectedness: relation to connectedness. Topologist's sign curve. Examples in R, R^2, R^3 etc. Connected subsets of R. Classification via connectedness **(12).** **Functional Analysis** Vector Spaces, Hamel basis, Infinite dimensional linear spaces, Basis of a linear space exists. Normed linear spaces (n.l.s.), examples of n.l.s. related to finite dimensional spaces, sequence spaces, space of continuous functions **(5).** The metric induced by norm. Convergence of sequence in n.l.s. and their properties, closure of a linear subspace is again a linear subspace**(3).** Banach space, examples and non-examples, complete subspaces and closed subspaces and their relations, completion of an n.l.s. Finite dimensional subspaces are closed but converse may not be true. Banach space cannot have a countably infinite basis. Separability of n.l.s**.(6**), Equivalent norms, examples and non-examples, all norms are equivalent on a finite dimensional space**(2).** Concept of convergent series in an n.l.s., an n.l.s. is complete if and only if every absolutely convergent series is convergent**(2).** Riesz lemma, closed and bounded subsets of an n.l.s. are compact if and only if the space is finite dimensional**(2).** Linear transformations, continuous linear transformations between n.l.s., examples and non- examples. Equivalent criterion of continuity of linear transformation. Bounded linear transformations, A linear transformation is continuous if and only if it is bounded. Linear functional, space of bounded linear transformations B(X,Y) as an n.l.s., unbounded linear transformations, examples , non-examples. On finite dimensional n.l.s. all linear transformations are bounded,

	computation of norm of linear functional, Y is complete if and only if B(X,Y) is complete.(8)
Texts	1. Topology: A First Course— J. R. Munkres 2. Functional Analysis with Applications: E. Kreyszig
Reading/Reference Lists	1. Introduction to General Topology- K.D.Joshi 2. Functional Analysis- Limaye
Evaluation	**CIA: 20** **End-Sem: 80**