Course: Discipline Specific Elective [Semester-6]

Semester	**6**
Paper Number	**Paper no:3 [HMTDS6031T]**
Paper Title	**Advanced Probability Theory and Statistics**
No. of Credits	**6**
Theory/ Composite	**Theory**
No of periods assigned	**Th: 6**
Name of Faculty Member(s)	**Prof. Sucharita Roy**
Course Description/ Objective	• To familiarize the students with the probability distribution of an n dimensional random vector through the n dimensional distribution function and studying its properties. • Introduction to functions of n dimensional random variables and finding their probability distributions either in form of joint p.m.f. or p.d.f. • Introduction of Covariance to study the interdependence between random variables and to study the connection between uncorrelated and independent random variables. • To introduce the idea of regression curves through conditional expectation and use principle of least squares to study regression lines, curves etc. • To introduce the concept of convergence of a sequence of random variables and study different forms of convergence, to get acquainted with the law of large numbers by dint of certain moment inequality and study Central –limit theorem along with applications. • To introduce the concept of sampling and sample (empirical) distribution function and to study sampling distributions of different statistics. • To introduce the idea of point as well as interval estimation and the theory of testing of hypotheses as an introductory exposition to statistical inference.
Syllabus	**Advanced Probability Theory and Statistics (78 classes)**

	Moments for univariate distributions. Raw and central. Properties [1]. Expectation, variance ,skewness and kurtosis and their interpretations [5]. Generating functions in one dimension. Moment generating function and characteristic functions.Properties.Examples [3]. Restrictions of probability generating function and moment generating function.Characterisation of distri bution function through characteristic function [3]. Two dimensional random variable. definition and examples[1], probability space for two dimensional random variable. joint distribution function. Properties.marginal distributions.[5] joint probability mass function and joint probability density function definition.[2] Properties. bivariate normal and uniform distributions.[3] Conditional distribution functions for discrete and continuous random variables.[2] Independence of a finite sequence of random variables. Given joint density function the marginal densities are uniquely determined but the converse is not true. Pairwise and mutual independence and their inter relationship. Extension to an infinite collection of random variables.[2] Transformation of two-dimensional random variables. [discrete and continuous], problems.[3] Moments for jointly distributed random variables. Expectation. Properties.Covariance and correlation coefficient. Properties. Interpretations.[3] Conditional expectation. Properties. Principles of least squares.[2] Determination of linear regression equation. Residual variance, its interpretation.[1] Given a bivariate normal density the conditional densities are univariate normal with mean linear in one of the variable [on which conditioning is done] and constant variance.[2] Joint moment generating functions. determination of moments from joint m.g.f.The necessary and sufficient condition of independence of two random variables in terms of m.g.f.[1]

	Bounds on tail probability by moment inequalities. Particular case: Chebycheff’s inequality[proof included],problems.[1] Convergence of sequence of random variables and limit theorems.convergence in probability. convergence in distribution. Weak law of large numbers. [3] Random sampling, sample statistics (2), sampling distributions of certain sample characteristics (sample mean and sample variance), moments of sample characteristics (2), exact sampling distributions: Chi-square, t-, and Fdistributions (3). Point estimation of a population characteristic or parameter, unbiased and consistent estimates (4), sample characteristics as estimates of the corresponding population characteristics (2), maximum likelihood estimates, application to Binomial, Poisson and Normal populations (4). Interval estimation: Confidence intervals or confidence limits for mean and standard deviation of a normal population (3). Approximate confidence limits for the parameter of a binomial population (1). Statistical hypothesis, simple and composite hypothesis, critical region of a test, type-I and type-II error, power function of a test, best critical region of a test (4), Neyman- Pearson theorem (1) and its application to Normal population (2), likelihood ratio testing and its application to Normal population (tests on mean and standard deviation of a normal population) (4), approximate tests on the parameter of a binomial population (1), simple problems of hypothesis testing (2).
Texts	1. Mathematical Probability: Banerjee, De, Sen 2. Mathematical Statistics: Banerjee, De, Sen.
Reading/Reference Lists	1. Basic Probability Theory: Robert B-Ash 2. Introduction to Probability Theory: Feller Vol.1

	3. Introduction to Probability Theory: Sheldon Ross 4. Modern Probability Theory: B.R.Bhatt 5. Introduction to Probability Theory: Parzen 6. Introduction to Mathematical Statistics: Hogg and Craig 7. Introduction to Statistical Theory: Mood, Graybill, Boes 8. Mathematical probability and Statistics: V. K. Rohatgi
Evaluation	**CIA:20** **End Sem:80**