

Semester	ONE
Paper Number	3
Paper Code	MDTS 4113
Paper Title	Linear Algebra and Elements of Statistical Inference
No. of Credits	6
Course description	<p>CORE Composite Paper Module 1: 2 classes/week Module 2: 2classes/week No. of classes assigned Theory: 4 classes per week Practical: 4 classes per week</p>
Course Objective	<p>After completion of the course a student is expected to have an idea of</p> <ul style="list-style-type: none"> • Matrix algebra and determinants. • Vector spaces, subspaces, their dimensions and basis. • Projection of vectors, orthogonalisation, systems of linear equations, different factorization techniques. • Characteristic roots and vectors along with the understanding of classification of quadratic forms. • Applications on least squares and dimension reduction. • This course is a pre requisite to the courses like Multivariate and Machine learning etc. • Understand the concept of an iid sample. • Conceptualise drawing samples from theoretical distributions. • Conceptualise level, size, power of a test and the errors associated with a testing problem. • Applying the results of sampling distributions to build test statistics and critical regions. • Construct confidence intervals for parameters and develop their relation with testing of hypotheses.

Syllabus

Module1: Linear Algebra

Unit 1: Vectors – Concept of a vector, length of a vector, Angle between two vectors, Orthogonal and orthonormal vectors, Linear dependence and independence of vectors, Vector spaces, Spanning set of a vector space, Basis of a vector space, Dimension of a vector space, Projection of a vector on a vector space, Orthogonal Basis, Orthocomplement of a vector space, Gram-Schmidt orthogonalization procedure, Row space & column space of a matrix.

Matrices (as a vector of vectors), Square matrices, Matrix operations (Addition, subtraction, multiplication by a scalar and by a matrix, Kronecker Product), Null matrix, Identity matrix, symmetric and skew symmetric matrices, orthogonal matrices, Rank of a matrix, singular and non-singular matrices, A few important results on the rank of a matrix, Inverse of a matrix, idempotent matrices, Elementary Transformations on a matrix, Reduction of a matrix to echelon, and diagonal forms by elementary transformations, Trace of a matrix, Partitioning of matrices and simple properties. [11]

Unit2: Direct and iterative methods for solving a linear system of equations: Gaussian elimination, LU factorization, QR factorization, Cholesky method, Jacobi's method, Gauss-Seidel method. [5]

Characteristic roots and Characteristic vector, Properties, Spectral Decomposition, Singular value decomposition. [5]

Quadratic forms: Classification & canonical reduction. [3]

	<p>Module 2 : Elements of Statistical Inference</p> <p>Unit 3: Sampling Distributions <i>Basic Concepts:</i> Concept of an iid sample, Statistic and its standard error. Drawing of random samples from theoretical distributions. Illustrations with R. (3) <i>Techniques of Sampling Distributions:</i> Distribution Function, Moment Generating Function and Transformation of variables technique to obtain sampling distribution of statistics. (2) <i>Basic Sampling Distributions from Univariate Normal Distributions:</i> Chi-square, t and F. Degrees of freedom. Sampling Distributions of sample mean, sample variance, their independence, linear combinations of normal variables. Definitions of Non-central chi-square, t and F distributions. Illustrations through simulations. Illustrations through simulations. (5) <i>Basic Sampling Distributions from Bivariate Normal Distributions:</i> Sampling distributions of sample correlation coefficient and linear regression coefficients. Illustrations through simulations. (2)</p> <p>Unit 4: Elements of Inference Problems <i>Problems and Paradigms of Inference:</i> Estimation and Testing of Hypotheses Problems. Parametric and Nonparametric Inference. Classical and Bayesian Inference. (3) <i>Estimation:</i> Basic Criteria of a good estimator – Sufficiency, Unbiasedness, Minimum Variance, Consistency and Efficiency. OPEF. (4) <i>Interval Estimation:</i> Methods of finding confidence intervals. Shortest confidence intervals. Confidence belts. (3) <i>Testing of Hypothesis:</i> Null and Alternative Hypothesis. Simple and Composite Hypothesis. Type-1 and Type-2 Errors. Level and Power of a Test. Power Function. p-value of a test. p-hacking. Application to tests of significance. [4]</p>
List of Practical	Based on the theory topics

Reading/Reference Lists	<ol style="list-style-type: none"> 1. Stephen Boyd and Lieven Vandenberghe, Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares (Cambridge University Press, 3rd edition) 2. Lloyd N. Trefethen and David Bau, III: Numerical linear algebra, SIAM (1997) 3. Matrix Algebra: Theory, Computations and Applications in Statistics by J.E. Gentle, Springer, 2007 4. Fundamentals of Matrix Computations by D.S. Watkins, 2nd ed., Wiley, New York, 2002. 5. Gilbert Strang; Linear Algebra and its Applications; Academic Press; Second Edition. 6. Goon A.M., Gupta M.K., Das Gupta.B.: Fundamentals of Statistics, Vol. 1, World Press, 2010. 7. Goon, A.M. Gupta, M.K. and Dasgupta, B. : An outline of Statistical Theory, Vol. 1, World Press, 2010. 8. Ismay, C. and Kim, A.Y., Statistical Inference via Data Science, A Modern Dive into R and the Tidyverse, CRC Press Talor and Francis group, 2020. 9. Moulin, P. and Venugopal, V.V., Statistical Inference for Engineers and Data Scientists, Cambridge University Press. 10. Caffo, B., Statistical Inference for Data Science, Leanpub, 2016. 	
Evaluation	Theory CIA: 10 End Sem Exam: 50 (25+25) Total : 60	Practical Continuous Assessment: 30 End Sem Viva: 10 Total: 40
Paper Structure for End Semester Theory	Short questions: 5 marks each	Long questions: 10 marks each
Module I	1 out of 2	2 out of 3
Module II	1 out of 2	2 out of 3